Appendix A

Example on how to use a permutation-based approach in EDAs

Section 3.3.1 explains the use of permutation-based individual representations for EDAs, and Figure 3.1 shows a procedure to translate a permutation-based representation to the matching solution it symbolizes. This appendix illustrates this procedure as well as some problems to be taken into account with an example.

A.1 Example of translating from a permutation to the solution it symbolizes

In order to demonstrate the representation of individuals containing permutations and the procedure for translating them to a point in the search space, we make use of the example shown in Figure A.1. In this example we are considering an inexact graph matching problem with a data graph G_D of 10 vertices ($|V_D| = 10$) and a model graph G_M of 6 vertices ($|V_M| = 6$). We also use a similarity measure for the example (the $\varpi(i, j)$ function), the results of which are shown in the same figure. This similarity function does not always have to be symmetrical, and just as an example of this we have chosen a non-symmetrical one (see Section 3.3.1 for a discussion on this topic). The translation has to produce individuals of the same size (10 variables, that is, 10 nodes in the Bayesian network), but each of their variables may contain a value between 1 and 6, that is, the number of the vertices of V_M with which any vertex of G_D can be matched in the solution.

Figure 3.1 shows the procedure for both phases 1 and 2. Following the procedure for phase 1, the first 6 vertices will be matched, and we will obtain the first matches for the three individuals in Figure A.1.

In the second phase, generation of the solution will be completed by processing one by one all the remaining variables of the individual. For that, we will consider the next variable that is still not treated, the 7^{th} in our example. Here, the first individual in the example has the value 7 in its 7^{th} variable (i.e. position), which means that vertex 7 of G_D will be matched next. Similarly, the vertices of G_D to be assigned to the 7^{th} position for the other two example individuals are vertices 10 and 4 respectively.

Next, in order to decide which vertex of G_M we have to assign to our vertex 7 of G_D

Individuals:

1	2	3	4	5	6	7	8	9	10

5	8	7	1	6	9	10	3	4	2

10	9	8	7	6	5	4	3	2	1

Similarity Function:

$\varpi(i,j)$	1	2	3	4	5	6	7	8	9	10
1	1.00	0.87	0.67	0.80	0.77	0.48	0.88	0.80	0.75	0.89
2	0.03	1.00	0.96	0.13	0.73	0.90	0.15	0.66	0.74	0.92
3	0.20	0.42	1.00	0.63	0.05	0.22	0.20	0.51	0.31	0.50
4	0.52	0.50	0.88	1.00	0.49	0.88	0.08	0.91	0.38	0.47
5	0.19	0.90	0.85	0.71	1.00	0.15	0.24	0.51	0.97	0.80
6	0.47	0.87	0.67	0.80	0.77	1.00	0.88	0.80	0.75	0.87
7	0.03	0.96	0.35	0.13	0.73	0.90	1.00	0.66	0.74	0.92
8	0.20	0.42	0.93	0.63	0.05	0.22	0.20	1.00	0.31	0.50
9	0.52	0.50	0.89	0.53	0.49	0.88	0.08	0.91	1.00	0.47
10	0.19	0.90	0.85	0.71	0.18	0.15	0.24	0.51	0.97	1.00

Figure A.1: Example of three permutation-based individuals and a similarity measure $\varpi(i,j)$ between vertices of the data graph ($\forall i,j \in V_D$) for a data graph with $|V_D| = 10$.

1	2	3	4	5	6	7	8	9	10
1	2	3	4	5	6	–	–	–	–

1	2	3	4	5	6	7	8	9	10
4	–	–	–	1	5	3	2	6	–

1	2	3	4	5	6	7	8	9	10
–	–	–	–	6	5	4	3	2	1

Figure A.2: Result of the generation of the individual after the completion of phase 1 for the example in Figure A.1 with G_D containing 6 vertices ($|V_M| = 6$).

according to the first individual, we analyze the similarity of vertex 7 of V_D and each of the previously matched vertices of G_D (vertex 1 to 6). This similarity measure is given by the function ϖ shown in Figure A.1. If we look at the 7^{th} line in this table we see that in columns 1 to 6, the highest value is 0.96, in column 2. Therefore, following the algorithm in phase 2, we match to vertex 7 of G_D the same vertex of G_M assigned to vertex 2 of G_D. As we can see in Figure A.2, for the first individual, vertex 2 in G_D was matched to the vertex 2 of G_M, and therefore we will also assign vertex 2 of G_M to the 7^{th} vertex of G_D.

Similarly, for the second individual, the 7^{th} variable of the individual is also processed. This has the value 10, so vertex 10 of G_D is therefore the next to be matched. We will compare this vertex with the vertices matched previously, i.e. vertices 5, 8, 7, 1, 6 and 9. The highest similarity value for these is $\varpi = 0.97$, in column 9. Therefore the most similar vertex is 9, and vertex 10 of G_D will be matched to the same vertex of G_M as vertex 9 of G_D was. Looking at Figure A.2, this is 6^{th} vertex of G_M. Following the same process for the third individual, we obtain that vertex 4 of G_D is matched with vertex 3 of G_M. Figure A.3 shows the result of this first step of phase 2.

Continuing this procedure of phase 2 until the last variable, we obtain the solutions shown in Figure A.4.

1	2	3	4	5	6	2	–	–	–
1	2	3	4	5	6	7	8	9	10

4	–	–	–	1	5	3	2	6	6
1	2	3	4	5	6	7	8	9	10

–	–	–	3	6	5	4	3	2	1
1	2	3	4	5	6	7	8	9	10

Figure A.3: Generation of the solutions for the example individuals in Figure A.1 after the first step of phase 2 ($|V_M| = 6$).

1	2	3	4	5	6	2	3	3	3
1	2	3	4	5	6	7	8	9	10

4	2	2	2	1	5	3	2	6	6
1	2	3	4	5	6	7	8	9	10

1	3	3	3	6	5	4	3	2	1
1	2	3	4	5	6	7	8	9	10

Figure A.4: Result of the generation of the solutions after the completion of phase 2.

Note that each of the vertices of G_D is assigned to a variable between 1 and $|V_M| = 6$. Note also that every vertex of G_M is matched to at least one vertex of G_D, and that a value is given to every vertex of G_D, giving a matching value to each of the segments in the data image (all the segments in the data image are therefore recognized with a structure of the model).

A.2 The redundancy problem on permutation-based representations

An important aspect of this individual representation based on permutations is that the cardinality of the search space is $n!$ This cardinality is higher than that of the traditional individual representation, but it is tested for its use with EDAs in graph matching for the first time here. In addition, it is important to note that a permutation-based approach can create redundancy in the solutions, as two different permutations may correspond to the same solution. An example of this is shown in Figure A.5, where two individuals with different permutations are shown and the solution they represent is exactly the same.

Individual 1:

1	2	3	4	5	6	7	8	9	10

Individual 2:

1	2	3	4	5	6	7	9	8	10

Solution they represent:

1	2	3	4	5	6	2	3	3	3
1	2	3	4	5	6	7	8	9	10

Figure A.5: Example of redundancy in the permutation-based approach. The two individuals represent the same solution shown at the bottom of the figure.