

Solving Consensus using S (worst case scenario)

5 processes: P1, P2, P3, P4, P5

No process crashes: strong completeness is (trivially) satisfied

P5 is never suspected: eventual weak accuracy is satisfied

Initially:

$V1 = v1$	\perp	\perp	\perp	\perp	$\Delta1 = v1$	\perp	\perp	\perp	\perp
$V2 = \perp$	$v2$	\perp	\perp	\perp	$\Delta2 = \perp$	$v2$	\perp	\perp	\perp
$V3 = \perp$	\perp	$v3$	\perp	\perp	$\Delta3 = \perp$	\perp	$v3$	\perp	\perp
$V4 = \perp$	\perp	\perp	$v4$	\perp	$\Delta4 = \perp$	\perp	\perp	$v4$	\perp
$V5 = \perp$	\perp	\perp	\perp	$v5$	$\Delta5 = \perp$	\perp	\perp	\perp	$v5$

$r = 1$

P1-P2-P3-P4 receive $\Delta5$ from P5: they all learn $v5$

P5 receives $\Delta4$ from P4: it learns $v4$

P4 receives $\Delta3$ from P3: it learns $v3$

P3 receives $\Delta2$ from P2: it learns $v2$

P2 receives $\Delta1$ from P1: it learns $v1$

$V1 = v1$	\perp	\perp	\perp	$v5$	$\Delta1 = \perp$	\perp	\perp	\perp	$v5$
$V2 = v1$	$v2$	\perp	\perp	$v5$	$\Delta2 = v1$	\perp	\perp	\perp	$v5$
$V3 = \perp$	$v2$	$v3$	\perp	$v5$	$\Delta3 = \perp$	$v2$	\perp	\perp	$v5$
$V4 = \perp$	\perp	$v3$	$v4$	$v5$	$\Delta4 = \perp$	\perp	$v3$	\perp	$v5$
$V5 = \perp$	\perp	\perp	$v4$	$v5$	$\Delta5 = \perp$	\perp	\perp	$v4$	\perp

P5 has some information ($v4$) that P1-P2-P3 do not have

If we go directly to Phase 2, P1-P2-P3 will decide $v5$, but P4-P5 will decide $v4$

$r = 2$

P1-P2-P3-P4 receive $\Delta5$ from P5: P1-P2-P3 learn $v4$

P5 receives $\Delta4$ from P4: it learns $v3$

P4 receives $\Delta3$ from P3: it learns $v2$

P3 receives $\Delta2$ from P2: it learns $v1$

$V1 = v1$	\perp	\perp	$v4$	$v5$	$\Delta1 = \perp$	\perp	\perp	$v4$	\perp
$V2 = v1$	$v2$	\perp	$v4$	$v5$	$\Delta2 = \perp$	\perp	\perp	$v4$	\perp
$V3 = v1$	$v2$	$v3$	$v4$	$v5$	$\Delta3 = v1$	\perp	\perp	$v4$	\perp
$V4 = \perp$	$v2$	$v3$	$v4$	$v5$	$\Delta4 = \perp$	$v2$	\perp	\perp	\perp
$V5 = \perp$	\perp	$v3$	$v4$	$v5$	$\Delta5 = \perp$	\perp	$v3$	\perp	\perp

P5 has some information ($v3$) that P1-P2 do not have

If we go directly to Phase 2, P1-P2 will decide $v4$, but P3-P4-P5 will decide $v3$

$r = 3$

P1-P2-P3-P4 receive Δ_5 from P5: P1-P2 learn v_3

P5 receives Δ_4 from P4: it learns v_2

P4 receives Δ_3 from P3: it learns v_1

$V_1 = v_1 \perp v_3 v_4 v_5$	$\Delta_1 = \perp \perp v_3 \perp \perp$
$V_2 = v_1 v_2 v_3 v_4 v_5$	$\Delta_2 = \perp \perp v_3 \perp \perp$
$V_3 = v_1 v_2 v_3 v_4 v_5$	$\Delta_3 = \perp \perp \perp \perp \perp$
$V_4 = v_1 v_2 v_3 v_4 v_5$	$\Delta_4 = v_1 \perp \perp \perp \perp$
$V_5 = \perp v_2 v_3 v_4 v_5$	$\Delta_5 = \perp v_2 \perp \perp \perp$

P5 has some information (v_2) that P1 does not have

If we go directly to Phase 2, P1 will decide v_3 , but P2-P3-P4-P5 will decide v_2

$r = 4$

P1-P2-P3-P4 receive Δ_5 from P5: P1 learns v_2

If P5 receives Δ_4 from P4, it learns v_1 , else it does not learn anything

$V_1 = v_1 v_2 v_3 v_4 v_5$	$\Delta_1 = \perp v_2 \perp \perp \perp$
$V_2 = v_1 v_2 v_3 v_4 v_5$	$\Delta_2 = \perp \perp \perp \perp \perp$
$V_3 = v_1 v_2 v_3 v_4 v_5$	$\Delta_3 = \perp \perp \perp \perp \perp$
$V_4 = v_1 v_2 v_3 v_4 v_5$	$\Delta_4 = \perp \perp \perp \perp \perp$
$V_5 = \perp v_2 v_3 v_4 v_5$	$\Delta_5 = \perp \perp \perp \perp \perp$

Now all processes have as much information as P5, so we can go to Phase 2

In Phase 2, processes exchange their V vectors. As they will at least receive V_5 from P5, P1-P2-P3-P4 will 'forget' the first component of their vectors

$V_1 = \perp v_2 v_3 v_4 v_5$
$V_2 = \perp v_2 v_3 v_4 v_5$
$V_3 = \perp v_2 v_3 v_4 v_5$
$V_4 = \perp v_2 v_3 v_4 v_5$
$V_5 = \perp v_2 v_3 v_4 v_5$

In Phase 3, the unanimous decision will be v_2

(Note: if P5 received Δ_4 from P4 in round 4, the decision would be v_1)