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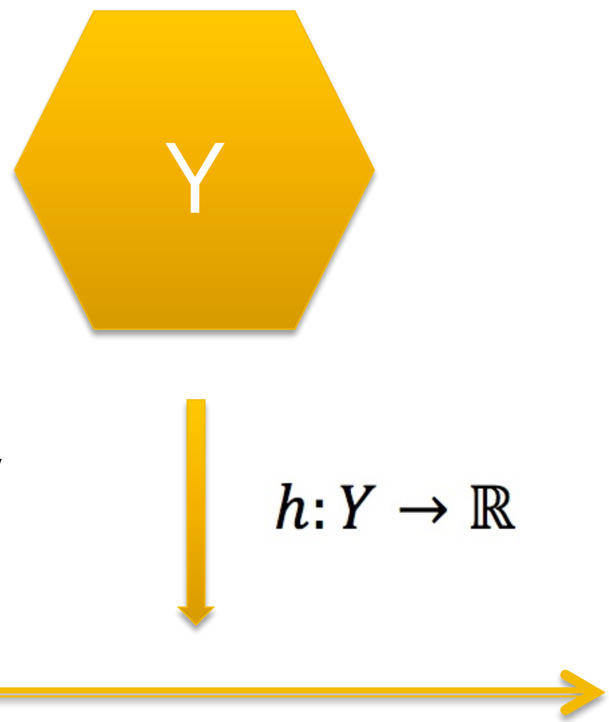
An Analysis of Indirect Optimisation Strategies for Scheduling

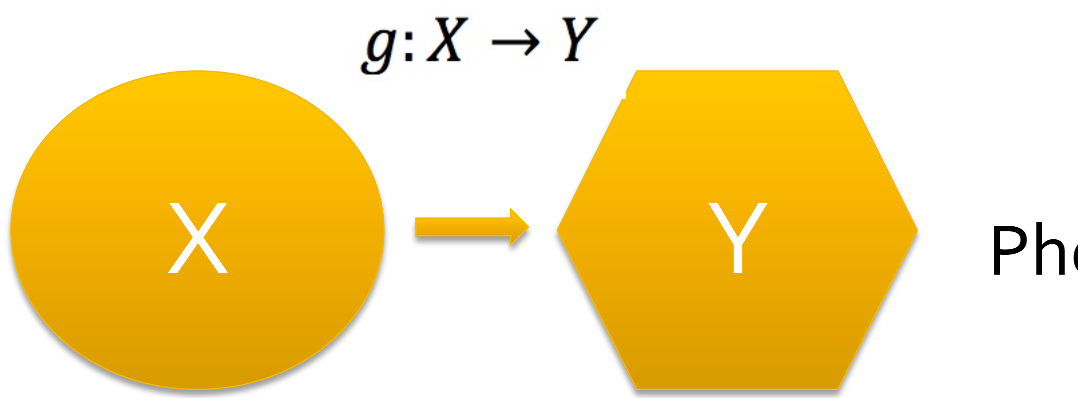
Search problem (Y,h)



Y may be irregular, difficult to search

Indirect Encoding

Genotype

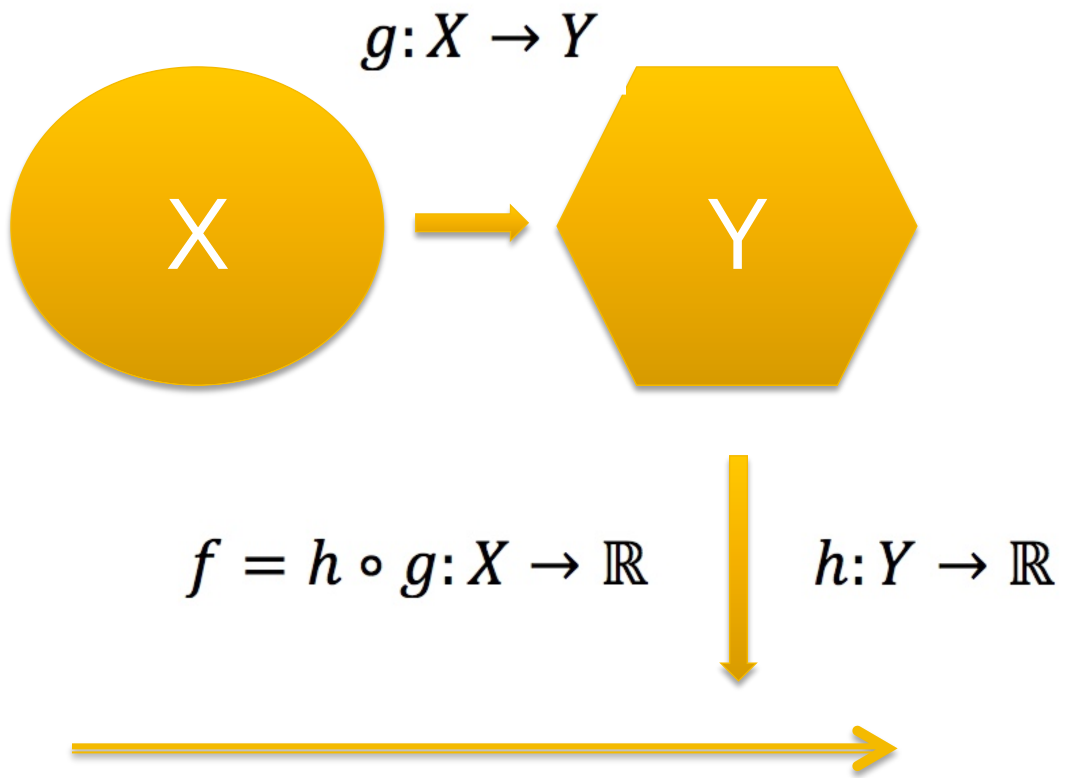


Phenotype

Decoder, g, may contain varying amounts of problem knowledge

$h: Y \to \mathbb{R}$

Indirect Encoding



Indirect Encodings

- A decoder, g: X → Y, maps search problems on Y into search problems on X
  - g: (Y,h) → (X,f)
- Algorithm spends time searching X and decoding.
- **Aim**: explore trade-offs between search and decoding costs
- **Focus**: permutation problems with greedy decoders

Problem Generation

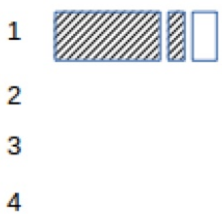
- Schedule n jobs on m machines
  - Minimise overall completion time: $\max_i C_i$
- All machines handle precisely k jobs $k \in [1, \frac{n}{m}] \cap \mathbb{Z}$
- Base time for job i on machine j is $P_{i,j}$
  - **Instance generator** for matrices P {n,m,μ,α,β}
- Job time on machine increased by a proportion of P_{ij}, **depending on running order**

$$C_i = \begin{cases} \sum_{j,k} P_{i,j} * (1 + \frac{1}{9+k}) * x_{i,j,k} & k = 1 \\ \sum_{j,k} (P_{i,j} * (1 + \frac{1}{9+k}) + \sum_{i'} C_{i'} * x_{i',j,k-1}) * x_{i,j,k} & k > 1 \end{cases}$$

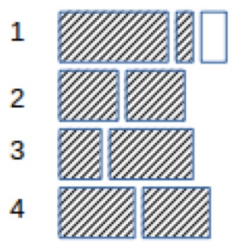
The Approach

1. Select decoders
2. Evaluate decoders to understand costs
3. Select search algorithms
4. Compare search algorithm performance
5. Analyse cost benefit trade offs

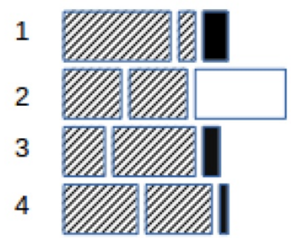
Greedy Decoders



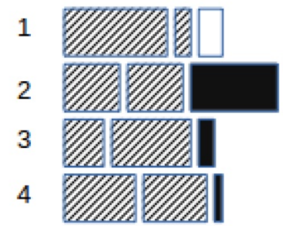
Length Greedy



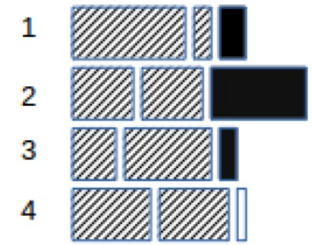
Width Greedy



Start Greedy



Finish Greedy



Evaluation Greedy

- Greedy decoders transform a permutation to a schedule, to be evaluated.

Algorithm 2 Overall greedy procedure to construct a schedule

Input permutation π
Initialise $x = 0_{n \times m \times n/m}$
for $i \leftarrow 1, |\pi|$ **do**
    $j, k \leftarrow$ **allocate**(π, i, x)
    $x_{\pi_i, j, k} = 1$
end for

Greedy Decoding Benefits

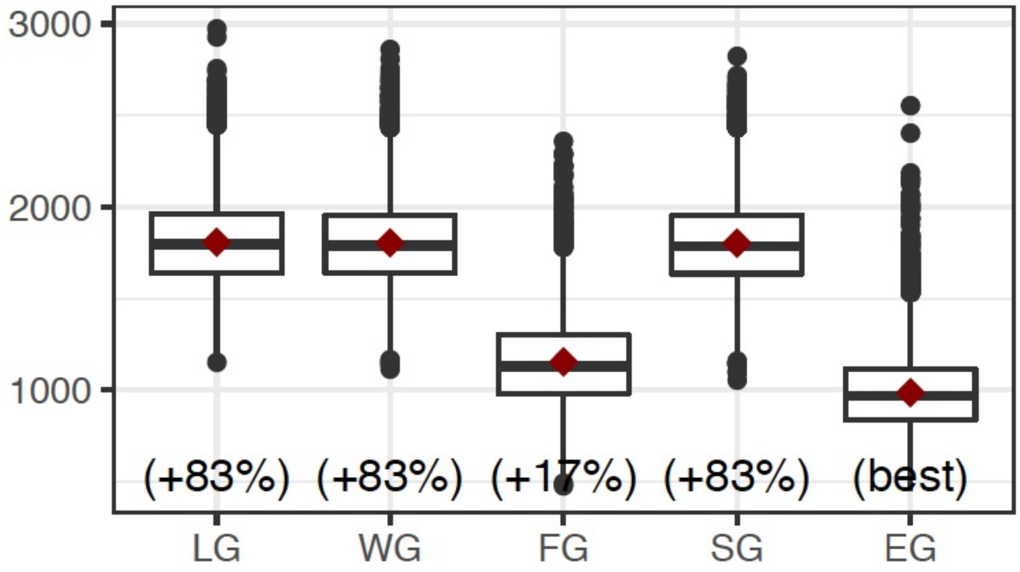


Fig. 8. Schedule quality using the five greedy search on instances generated using $\alpha = 0.9$ and $\beta = 0.9$

Greedy Decoding Costs

AVERAGE RUNTIME OF THE GREEDY PROCEDURES OVER 1000 RUNS ON 50X5 INSTANCES, SHOWING THE TIME (IN μS) REQUIRED TO PERFORM A SINGLE EVALUATION

Greedy Proc.	$\{\alpha, \beta\}$		
	$\{0.9, 0.1\}$	$\{0.9, 0.3\}$	$\{0.9, 0.9\}$
LG	320 (143)	308 (322)	310 (107)
WG	284 (108)	281 (77)	282 (119)
FG	1670 (335)	1678 (257)	1763 (294)
SG	1263 (232)	1241 (288)	1249 (242)
EG	1625 (247)	1501 (227)	1503 (283)

Search Algorithms

- Hill Climber (HC)
  - Random swap neighbourhoods
- Genetic Algorithm (GA)
  - Tournament selection (3) , PMX, elitism, P = 2n
- Mutations on Selection Algorithm (MOSA)
  - Population-based algorithm
  - Truncation selection and swap mutation
    - P = 2n, truncation = 5%
  - Multiple mutations around high value solutions
    - Somewhere "between" HC and GA

Experiments

- Combinatorial pairings of search algorithms and decoders
- Problem instances generated n = 20, 50, 100
  - For each n, m = n/2, m = n/10
  - 5 instances generated for each setting {n,m,α,β}
- 50 runs per algorithm per instance

TABLE II
MAXIMUM NUMBER OF FITNESS EVALUATIONS

n	Fitness Evaluations
20	1200
50	30000
100	60000

Results

TABLE III
RESULTS ON INSTANCES WITH $n = 20$

	GA	HC	MOSA
20-2-0.9-0.1-LG	1214.61	1240.62	1195.79
20-2-0.9-0.1-WG	1215.19	1241.20	**1191.31**
20-2-0.9-0.1-FG	1214.52	1229.22	**1191.24**
20-2-0.9-0.1-SG	1221.49	1238.42	**1191.24**
20-2-0.9-0.1-EG	1374.28	1378.90	**1191.24**
20-2-0.9-0.3-LG	1063.01	1139.31	1034.47
20-2-0.9-0.3-WG	1060.78	1134.85	**1032.45**
20-2-0.9-0.3-FG	1056.46	1080.99	**1032.45**
20-2-0.9-0.3-SG	1090.25	1127.78	**1032.45**
20-2-0.9-0.3-EG	1085.96	1095.11	**1032.45**
20-2-0.9-0.9-LG	1004.99	1217.20	**980.82**
20-2-0.9-0.9-WG	1004.15	1209.14	**980.44**
20-2-0.9-0.9-FG	988.42	1008.92	**980.44**
20-2-0.9-0.9-SG	1070.72	1191.26	**980.44**
20-2-0.9-0.9-EG	1036.64	1049.05	**980.44**
20-10-0.9-0.1-LG	300.82	320.82	282.13
20-10-0.9-0.1-WG	304.11	320.32	**279.70**
20-10-0.9-0.1-FG	288.68	309.26	**279.47**
20-10-0.9-0.1-SG	294.79	320.96	**279.47**
20-10-0.9-0.1-EG	295.09	308.46	**279.47**
20-10-0.9-0.3-LG	263.34	306.77	228.88
20-10-0.9-0.3-WG	268.20	306.49	228.11
20-10-0.9-0.3-FG	234.04	252.97	**226.33**
20-10-0.9-0.3-SG	264.58	304.10	227.85
20-10-0.9-0.3-EG	242.02	256.12	**225.92**
20-10-0.9-0.9-LG	219.37	326.46	129.83
20-10-0.9-0.9-WG	219.00	332.86	**126.19**
20-10-0.9-0.9-FG	132.74	145.78	**126.19**
20-10-0.9-0.9-SG	250.99	323.22	**126.19**
20-10-0.9-0.9-EG	132.87	153.23	**126.19**

TABLE IV
RESULTS ON INSTANCES WITH $n = 50$

	GA	HC	MOSA
50-5-0.9-0.1-LG	1229.50	1354.18	1193.07
50-5-0.9-0.1-WG	1231.71	1356.23	1187.84
50-5-0.9-0.1-FG	1234.16	1307.51	1186.88
50-5-0.9-0.1-SG	1259.92	1338.93	1186.88
50-5-0.9-0.1-EG	1196.14	1243.53	**1178.44**
50-5-0.9-0.3-LG	1003.92	1286.86	931.72
50-5-0.9-0.3-WG	1006.28	1282.26	**927.83**
50-5-0.9-0.3-FG	1000.12	1097.42	**927.29**
50-5-0.9-0.3-SG	1135.23	1257.80	**927.29**
50-5-0.9-0.3-EG	951.33	1032.39	**927.00**
50-5-0.9-0.9-LG	650.14	1295.53	562.57
50-5-0.9-0.9-WG	648.18	1293.98	**557.78**
50-5-0.9-0.9-FG	599.41	694.28	**556.61**
50-5-0.9-0.9-SG	1005.87	1246.13	**557.65**
50-5-0.9-0.9-EG	607.36	649.81	**556.37**
50-25-0.9-0.1-LG	356.62	419.99	**343.23**
50-25-0.9-0.1-WG	356.79	420.93	**343.04**
50-25-0.9-0.1-FG	347.05	399.01	**343.04**
50-25-0.9-0.1-SG	353.56	418.51	**343.04**
50-25-0.9-0.1-EG	357.59	397.11	**343.04**
50-25-0.9-0.3-LG	278.00	386.04	**245.33**
50-25-0.9-0.3-WG	279.85	386.55	**245.05**
50-25-0.9-0.3-FG	249.58	304.11	**245.05**
50-25-0.9-0.3-SG	284.63	386.29	**245.05**
50-25-0.9-0.3-EG	251.92	288.35	**245.05**
50-25-0.9-0.9-LG	237.00	439.00	**204.28**
50-25-0.9-0.9-WG	233.52	441.50	**204.25**
50-25-0.9-0.9-FG	**204.44**	209.27	**204.25**
50-25-0.9-0.9-SG	295.07	434.40	**204.25**
50-25-0.9-0.9-EG	**204.49**	208.86	**204.25**

TABLE V
RESULTS ON INSTANCES WITH $n = 100$

	GA	HC	MOSA
100-10-0.9-0.1-LG	1120.27	1305.59	1056.85
100-10-0.9-0.1-WG	1123.58	1305.80	1053.83
100-10-0.9-0.1-FG	1107.68	1228.34	1052.38
100-10-0.9-0.1-SG	1166.01	1284.96	1052.38
100-10-0.9-0.1-EG	1079.45	1194.90	**1036.69**
100-10-0.9-0.3-LG	963.99	1316.90	833.26
100-10-0.9-0.3-WG	971.99	1318.22	825.90
100-10-0.9-0.3-FG	884.45	1026.83	823.71
100-10-0.9-0.3-SG	1139.47	1284.78	825.21
100-10-0.9-0.3-EG	822.05	964.48	**789.34**
100-10-0.9-0.9-LG	747.47	1555.54	538.70
100-10-0.9-0.9-WG	755.05	1564.57	529.12
100-10-0.9-0.9-FG	547.37	699.35	497.05
100-10-0.9-0.9-SG	1309.65	1488.05	528.05
100-10-0.9-0.9-EG	522.48	605.02	**486.82**
100-50-0.9-0.1-LG	376.07	434.45	**358.19**
100-50-0.9-0.1-WG	378.49	434.91	**358.05**
100-50-0.9-0.1-FG	**358.62**	406.73	**358.05**
100-50-0.9-0.1-SG	366.42	435.37	**358.05**
100-50-0.9-0.1-EG	360.04	402.10	**358.05**
100-50-0.9-0.3-LG	347.68	429.19	**321.55**
100-50-0.9-0.3-WG	350.07	426.70	**321.19**
100-50-0.9-0.3-FG	**321.38**	341.20	**321.19**
100-50-0.9-0.3-SG	341.96	426.42	**321.19**
100-50-0.9-0.3-EG	**321.45**	332.74	**321.19**
100-50-0.9-0.9-LG	269.05	477.13	147.23
100-50-0.9-0.9-WG	264.82	478.36	140.50
100-50-0.9-0.9-FG	**137.03**	152.58	**136.59**
100-50-0.9-0.9-SG	320.00	475.25	140.25
100-50-0.9-0.9-EG	**137.52**	147.06	**136.59**

GA v MOSA

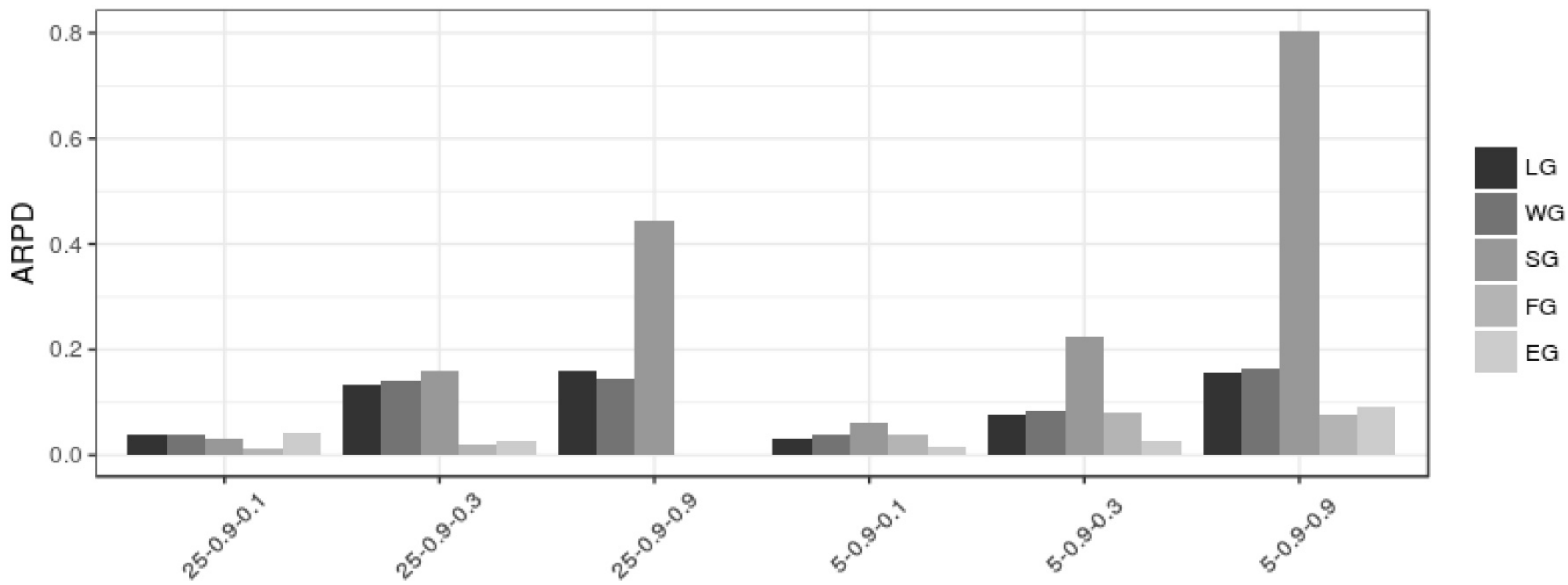


Fig. 11. ARPD of GA vs MOSA for combined with all greedy procedures on all instances of size n=50

Search-Decoder Pairings

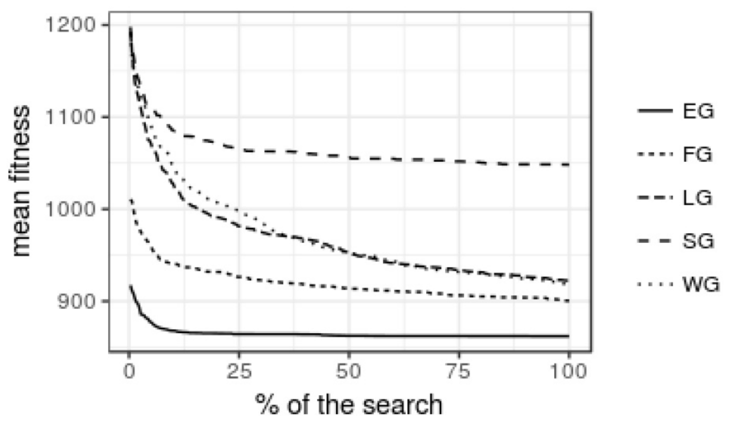


Fig. 9. Fitness of the best solution obtained by the GA over time, averaged over 20 runs on *50-5-0.9-0.3-1*

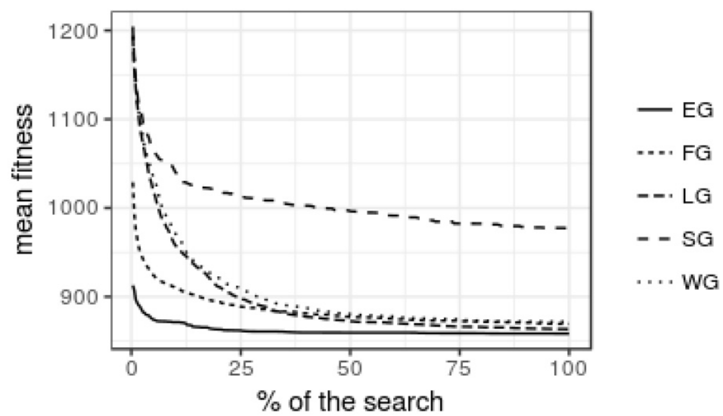


Fig. 10. Fitness of the best solution obtained by MOSA over time, averaged over 20 runs on *50-5-0.9-0.3-1*

GA

MOSA

Conclusions

- Population based selection in permutation space advantageous
  - GA, MOSA outperform HC
- Recombination not advantageous
  - MOSA outperforms GA
- Extra heuristics on the decoder can help all search algorithms
  - The wrong heuristic (SG) can be adverse
  - Decoder cost can outweigh additional benefits