Hill-Climbing Algorithm:
let's go for a walk before finding the optimum

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Objectives

- Analysis of the solutions found in the attraction basins: distance to the local optimum vs. number of steps of the algorithm.
- The paths defined by a hill-climbing algorithm do not monotonically reduce the distance to the local optimum.
- Visual examples of the paths built by the algorithm.

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Outline

1. Introduction
2. Results
3. Visualization
4. Conclusions

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Combinatorial Optimization Problem

Definition

A Combinatorial Optimization Problem consists of finding the points σ^* that minimize or maximize a function f:

$$\sigma^* = \arg\min_{\sigma \in \Omega} f(\sigma)$$

where Ω is a finite or countable infinite set

Permutation-based COP

A COP where Ω is the space of permutations of size n

- Permutation Flowshop Scheduling Problem
- Quadratic Assignment Problem
- Linear Ordering Problem

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n jobs
m machines
Each job consists of m operations

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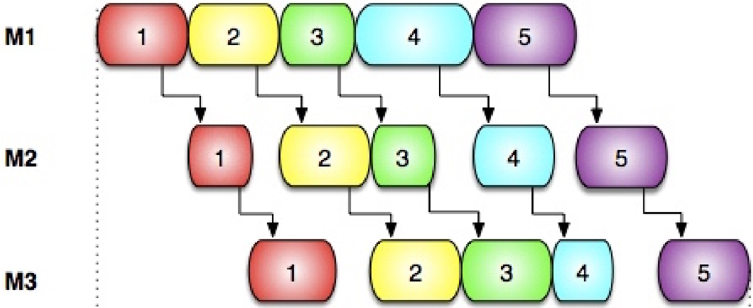
Each job consists of *m* operations



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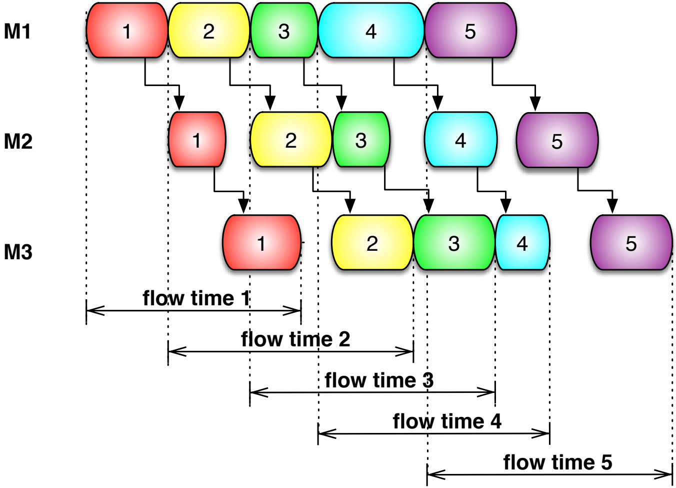
Permutation Flowshop Scheduling Problem



Total Flow Time = FT1 + FT2 + FT3 + FT4 + FT5

minimize TFT

Neighborhood

A neighborhood N in a search space Ω is a mapping that assigns a set of neighboring solutions $N(\sigma) \in \mathcal{P}(\Omega)$ to each solution $\sigma \in \Omega$:

$$\begin{array}{rccc} N: & \Omega & \longrightarrow & \mathcal{P}(\Omega) \\ & \sigma & \longmapsto & N(\sigma) \end{array}$$

Neighborhoods. Examples

2-exchange or Swap

Swap two items, not necessarily adjacent

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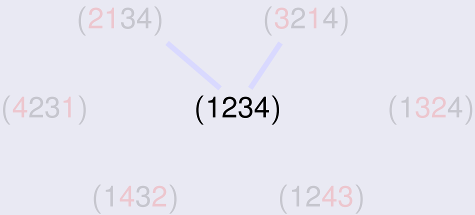

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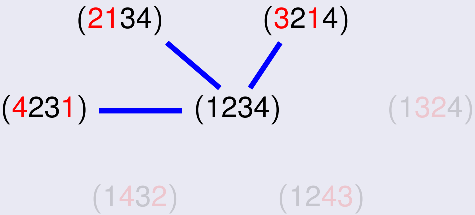

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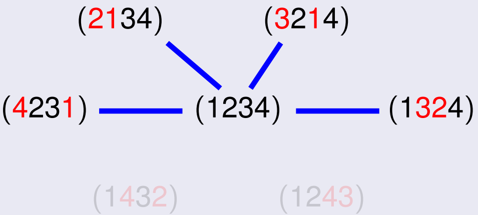

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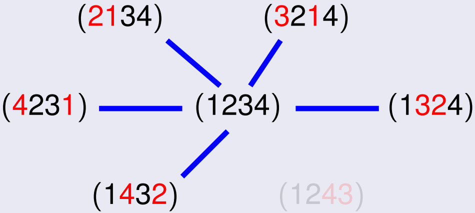

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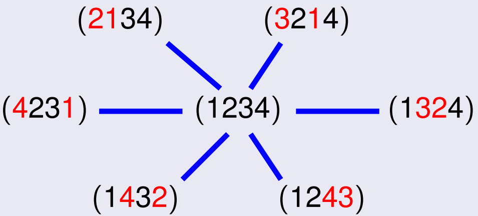

Neighborhoods. Examples

Insert

Move an item to a different position

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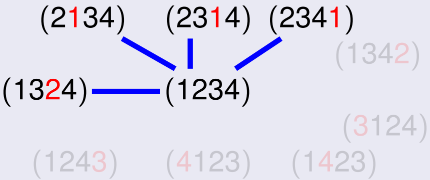

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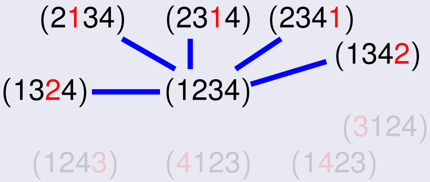

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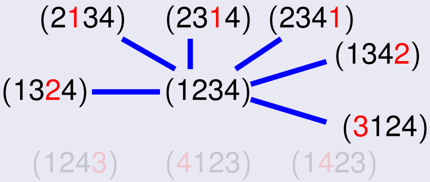

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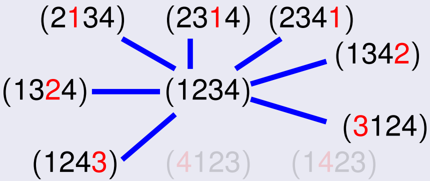

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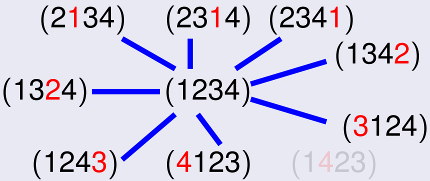

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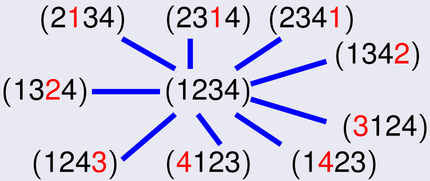

Distance

- σ_1 and σ_2 are at distance i if, starting from σ_1, and moving from neighboring to neighboring solutions, the length of the shortest path to reach σ_2 is i.
- Two neighboring permutations are at distance one.
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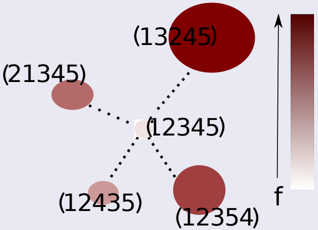
Local Optima

- A solution $\sigma^* \in \Omega$ is a local optimum if

$$f(\sigma^*) \leq f(\sigma),\ \forall \sigma \in N(\sigma^*)\ \text{(local minimum)}$$



Attraction basins of local optima

The attraction basin of a local optimum σ^*:

$$\mathcal{B}_{\sigma^*} = \{\sigma \in \Omega |\ \mathcal{H}(\sigma) = \sigma^*\},$$

where $\mathcal{H}$ is the operator that associates to each solution σ, the local optimum obtained after applying the algorithm

Deterministic best-improvement local search algorithm

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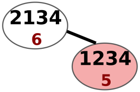

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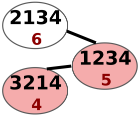

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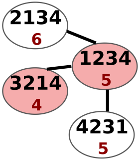

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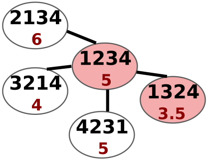

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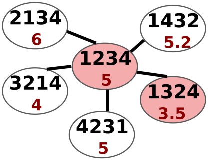

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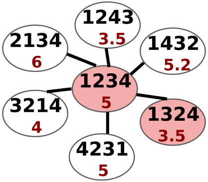

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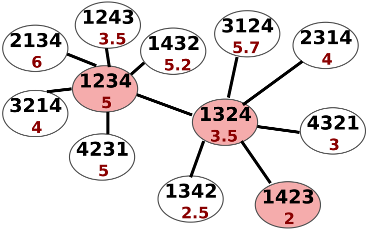

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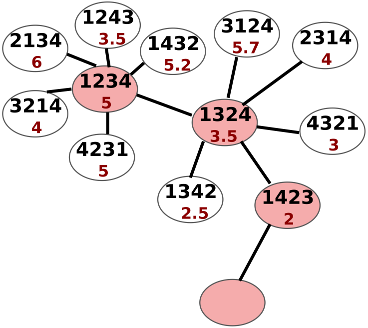

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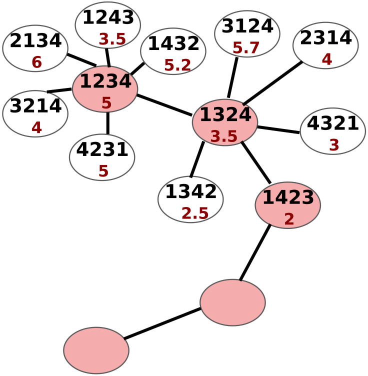

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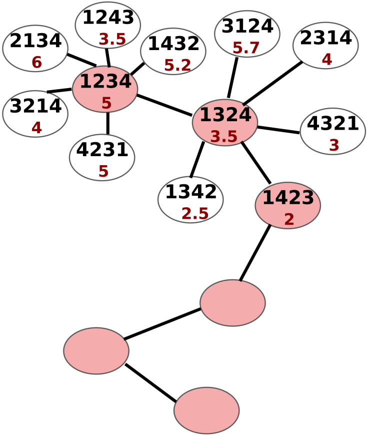

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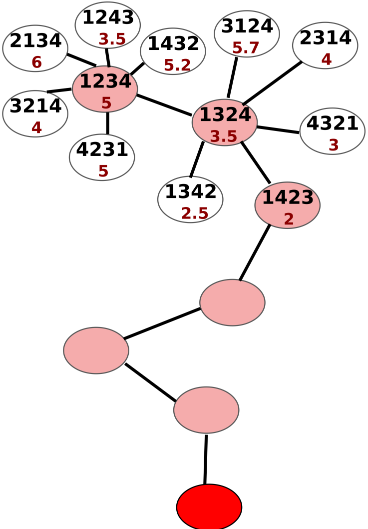

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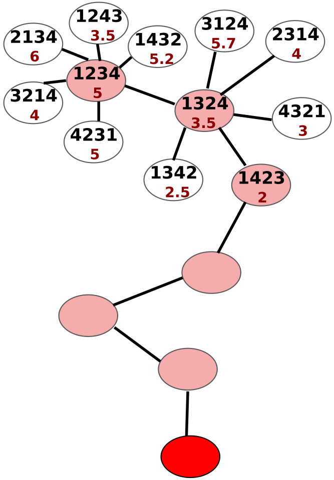

Choose an initial solution $\sigma \in \Omega$
repeat
  $\sigma^* = \sigma$
  for each $\sigma'_i \in \mathcal{N}(\sigma^*)$ **do**
    if $f(\sigma'_i) < f(\sigma)$ **then**
      $\sigma = \sigma'_i$
    end if
  end for
until $\sigma = \sigma^*$

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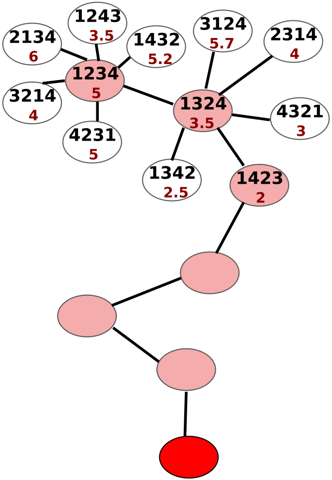

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The attraction basins are sets of paths!

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Experimental Design

- 9 PFSP instances of Taillard's benchmark.
- 10 jobs and 5 machines (n=10).
- The local optima and the attraction basins are calculated considering the 2-exchange and the insert neighborhoods.

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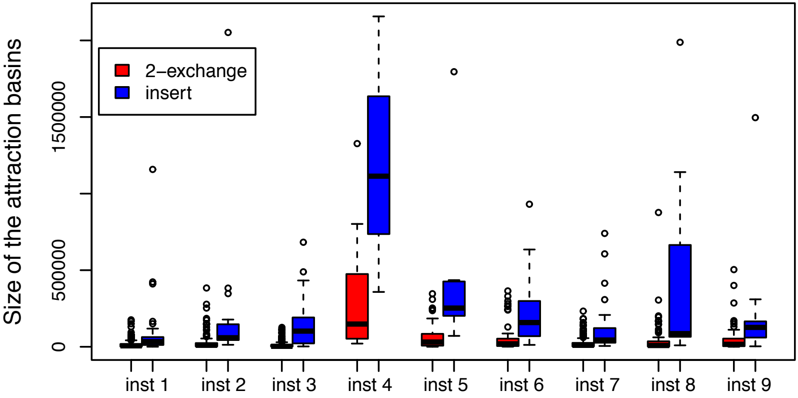
No. of local optima and sizes of attraction basins

# Loc. Opt	inst 1	inst 2	inst 3	inst 4	inst 5	inst 6	inst 7	inst 8	inst 9
2-exch.	225	117	295	11	58	58	158	83	80
insert	43	16	24	3	8	15	31	8	19



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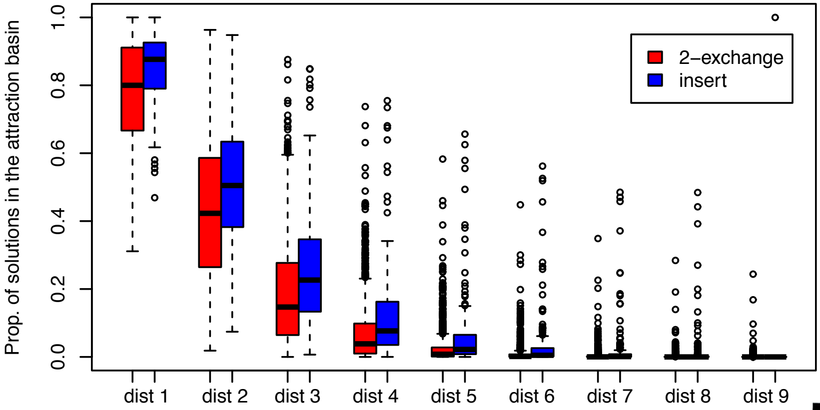
Distance to local optima vs. number of solutions in attraction basins



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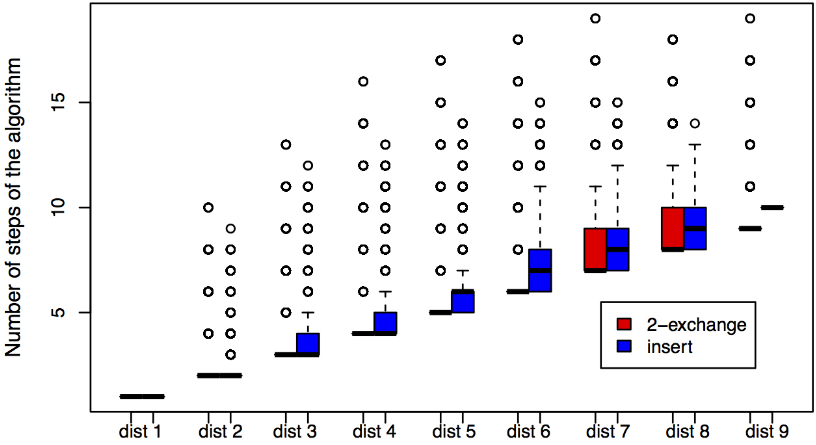
Distance to local optima vs. number of steps of the algorithm



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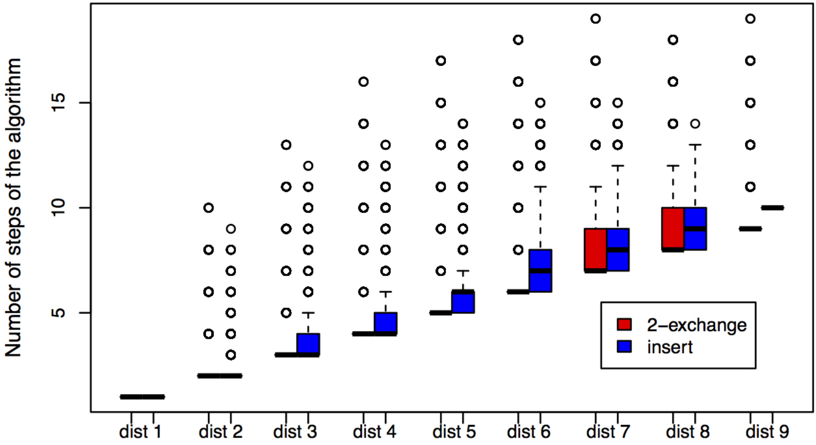

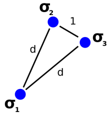

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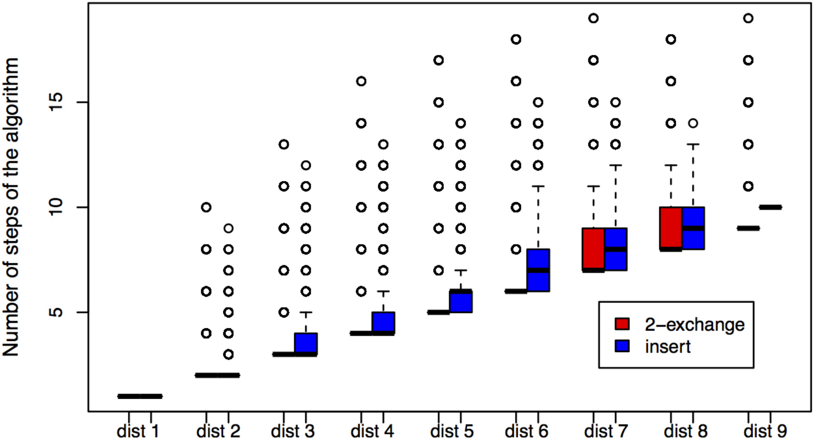

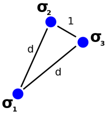

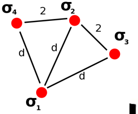

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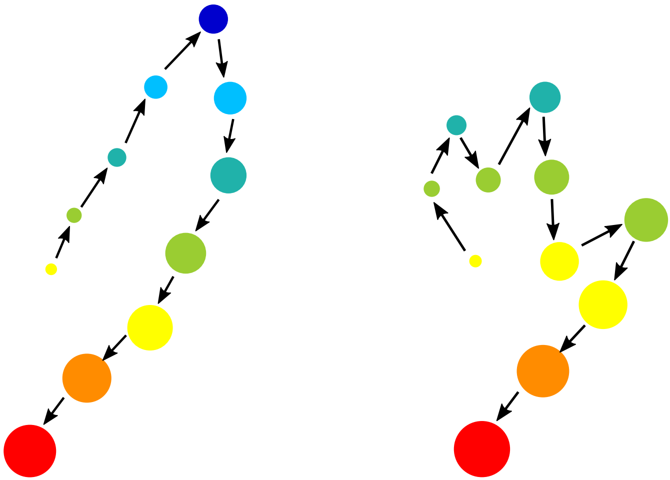
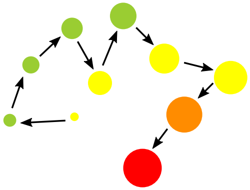
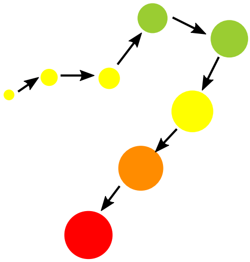
Visualization of paths: PFSP 2-exchange



Visualization of paths: PFSP insert





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Future Work

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