

Sailkapen-analisia

(cluster analysis)

Datu-analisiaren zenbait teknika

	Baieztatzailea Erabakitzailea <i>Gainbegiratua</i>	Miatzailea Deskribatzailea <i>Gainbegiratu gabea</i>
Lineala	Bariantz-analisia (ANOVA) Analisi bereizlea Erregresio-analisia	Faktore-analisia (osagai nagusiak (PCA), korrespondentziak, LSI, ...) Dimentsioanitzeko mailakatzea
Ez-lineala	Elkartze-arauak Segmentazioa (Erabakitze-zuhaitzak) (Sailkatze eta erregresio-zuhaitzak) <i>K</i> -auzokide gertuenak 'Support Vector Machine' (SVP)	Kontzeptuen sailkapena (COBWEB) Sailkapen automatikoa (‘Cluster analysis’) (Hierarkia, Partiketa (‘k-means’), ...)
	Neurona-sareak Probabilitate-sareak (bayestarrek)	

Sailkapen-analisia

(*cluster analysis*)

Sinonimoak

- ***Clustering*** (formen ezagutza, *pattern recognition*en arloan)
- *Sailkapen automatikoa* (*datu-analisia*n)
- *Sailkapen gainbegiratu gabea* (*estatistika*n, *datu-analisia*n)
- *Ikasketa gainbegiratu gabea* (*adimen artifiziale*n)
- *Segmentazioa* (*marketing*ean, *seinaleen analisia*n)
- *Vector quantization* (*seinaleen analisia*n)
- *Ordenazioa* (*psikologia*n)
- *Taxonomia numerikoa* (*biologia*n)
- *Analisi tipologikoa*

Sailkapen-analisia

(*cluster analysis*)

Definizioa (ez zehatza)

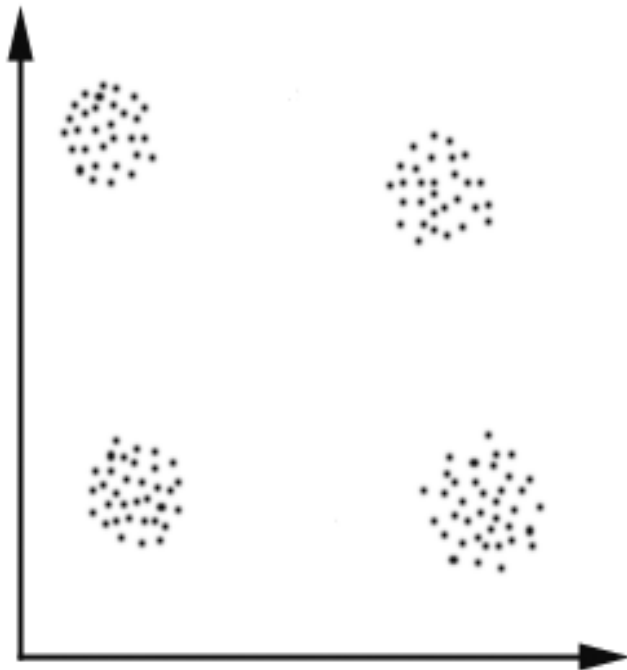
Sail edo *cluster* bat objektu-multzo bat da, zentzuren batean beraien artean antzekoak direnak, eta beste objektuekin alderatuta ezantzekoak.

Analisiaren **helburua**:

- Sail edo *cluster* horiek bilatu.
- Datuen multzoan *aldez aurretik* erezaguna den *egitura* bat bilatu.

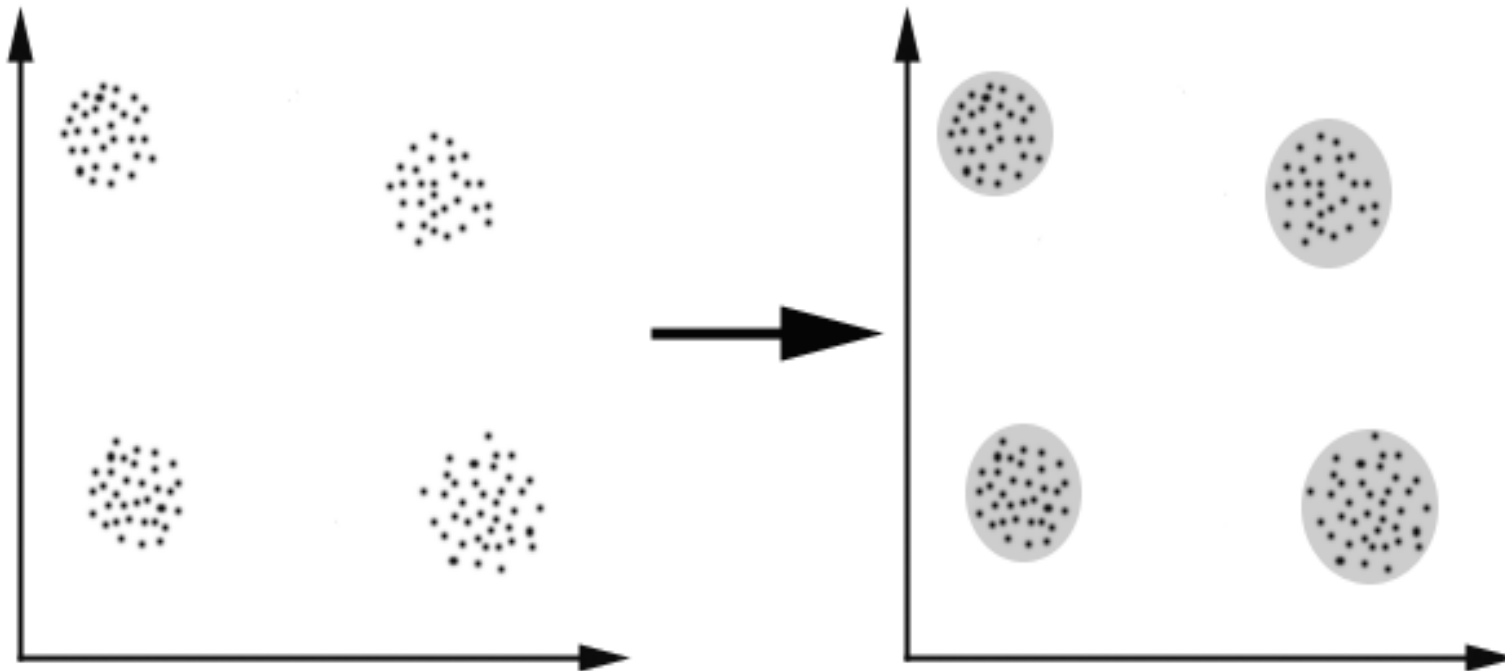
Sailkapen-analisia

(*cluster analysis*)



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Sailkapen-analisia (*cluster analysis*)

‘Berezko irakasbide bat egiteko
modu bakarra antzekoak diren
gauzak elkarrekin jartzea, eta
antzekoak ez direnak bereiztea da’.

Historia natural, 1749.



Georges-Louis Leclerc, comte de Buffon (1707-1788)

Sailkapen-analisisia

(cluster analysis)

Helburua

Konputagailu batean jarri gizakiok bi dimentsiotan sailak edo *clusterrak* **identifikatzeko** dugun gaitasuna, eta gero, gaitasun horiek dimentsio gehiagotara zabaldu.

Hasieran, zeregina erraza dela dirudi, baina praktikan zaila gertatzen da.

Sailkapen-analisia

(*cluster analysis*)

Helburua:

Sail edo *cluster* horiek bilatu.

Nola lortu helburua?

Sailkapen-analisisia

(cluster analysis)

Helburua:

Sail edo *cluster* horiek bilatu.

Nola lortu helburua?

Abiapuntua

Datuak (erlazioak):

Objektuak aldagai edo ezaugarri anitzen bitartez daude deskribatuta.

Sailkapen-analisia

(cluster analysis)

Ω , objektuen multzoa

X_j , aldagaiak

x_{ij} , ω_i objektuaren balioa X_j aldagaian

Ω	X_1		X_j		X_p
ω_1	x_{11}		x_{1j}		x_{1p}
ω_i	x_{i1}		x_{ij}		x_{ip}
ω_n	x_{n1}		x_{nj}		x_{np}

Suitza, 1888: 47 herrialde frankofono

Zenbait adierazle sozioekonomiko (%) eta emankortasuna

X1 Fertility X4 Education		X2 Agriculture X5 Catholic					X3 Examination X6 Infant.Mortality						
	X1	X2	X3	X4	X5	X6		X1	X2	X3	X4	X5	X6
Courtelay	80.2	17.0	15	12	9.96	22.2	Oron	72.5	71.2	12	1	2.40	21.0
Delemont	83.1	45.1	6	9	84.84	22.2	Payerne	74.2	58.1	14	8	5.23	23.8
Franches-Mnt	92.5	39.7	5	5	93.40	20.2	Paysd'enhaut	72.0	63.5	6	3	2.56	18.0
Moutier	85.8	36.5	12	7	33.77	20.3	Rolle	60.5	60.8	16	10	7.72	16.3
Neuveville	76.9	43.5	17	15	5.16	20.6	Vevey	58.3	26.8	25	19	18.46	20.9
Porrentruy	76.1	35.3	9	7	90.57	26.6	Yverdon	65.4	49.5	15	8	6.10	22.5
Broye	83.8	70.2	16	7	92.85	23.6	Conthey	75.5	85.9	3	2	99.71	15.1
Glane	92.4	67.8	14	8	97.16	24.9	Entremont	69.3	84.9	7	6	99.68	19.8
Gruyere	82.4	53.3	12	7	97.67	21.0	Herens	77.3	89.7	5	2	100.00	18.3
Sarine	82.9	45.2	16	13	91.38	24.4	Martigwy	70.5	78.2	12	6	98.96	19.4
Veveyse	87.1	64.5	14	6	98.61	24.5	Monthey	79.4	64.9	7	3	98.22	20.2
Aigle	64.1	62.0	21	12	8.52	16.5	St Maurice	65.0	75.9	9	9	99.06	17.8
Aubonne	66.9	67.5	14	7	2.27	19.1	Sierre	92.2	84.6	3	3	99.46	16.3
Avenches	68.9	60.7	19	12	4.43	22.7	Sion	79.3	63.1	13	13	96.83	18.1
Cossonay	61.7	69.3	22	5	2.82	18.7	Boudry	70.4	38.4	26	12	5.62	20.3
Echallens	68.3	72.6	18	2	24.20	21.2	La Chauxdfnd	65.7	7.7	29	11	13.79	20.5
Grandson	71.7	34.0	17	8	3.30	20.0	Le Locle	72.7	16.7	22	13	11.22	18.9
Lausanne	55.7	19.4	26	28	12.11	20.2	Neuchatel	64.4	17.6	35	32	16.92	23.0
La Vallee	54.3	15.2	31	20	2.15	10.8	Val de Ruz	77.6	37.6	15	7	4.97	20.0
Lavaux	65.1	73.0	19	9	2.84	20.0	ValdeTravers	67.6	18.7	25	7	8.65	19.5
Morges	65.5	59.8	22	10	5.23	18.0	V. De Geneve	35.0	1.2	37	53	42.34	18.0
Moudon	65.0	55.1	14	3	4.52	22.4	Rive Droite	44.7	46.6	16	29	50.43	18.2
Nyone	56.6	50.9	22	12	15.14	16.7	Rive Gauche	42.8	27.7	22	29	58.33	19.3
Orbe	57.4	54.1	20	6	4.20	15.3							

Sailkapen-analisia

(*cluster analysis*)

Helburua:

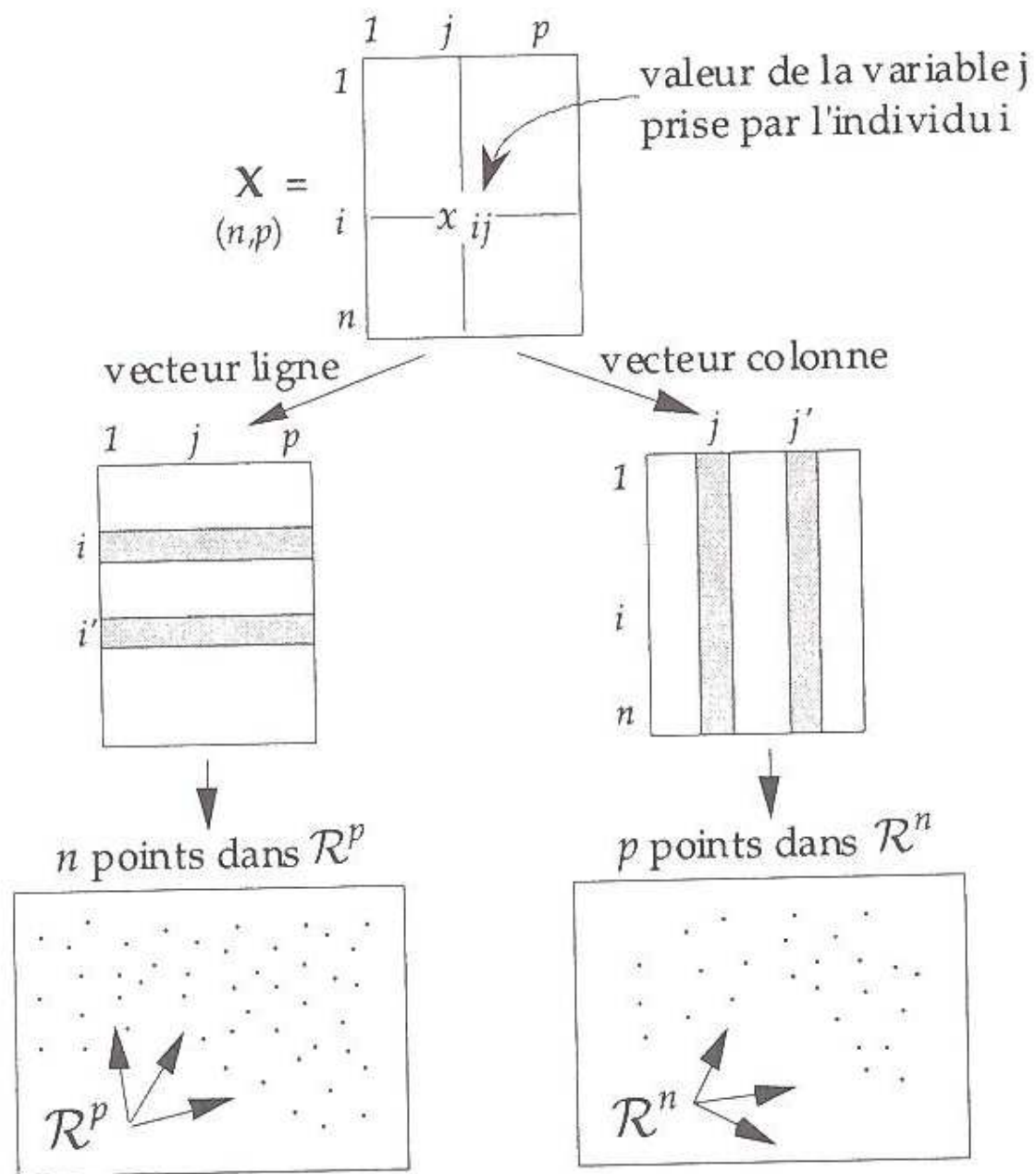
Sail edo *cluster* horiek bilatu.

Nola lortu helburua?

Objektuen **errepresentazio geometrikoa**:

- Objektu bakoitza puntu bat da dimentsio anitzeko espazio batean.
- Objektuen arteko ezantzekotasuna *distantzia* baten bitartez islatzen da.

Errepresentazio geometrikoa



Sailkapen-analisia

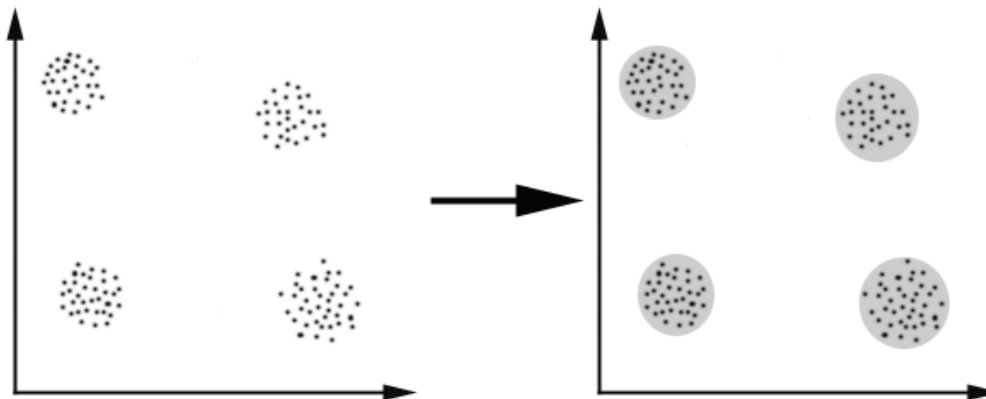
(*cluster analysis*)

Objektuen **errepresentazio** geometrikoa :

Objektuen arteko *distantzia*: bi objektu are eta antzekoagoak izan, orduan eta gertuago espazioan.

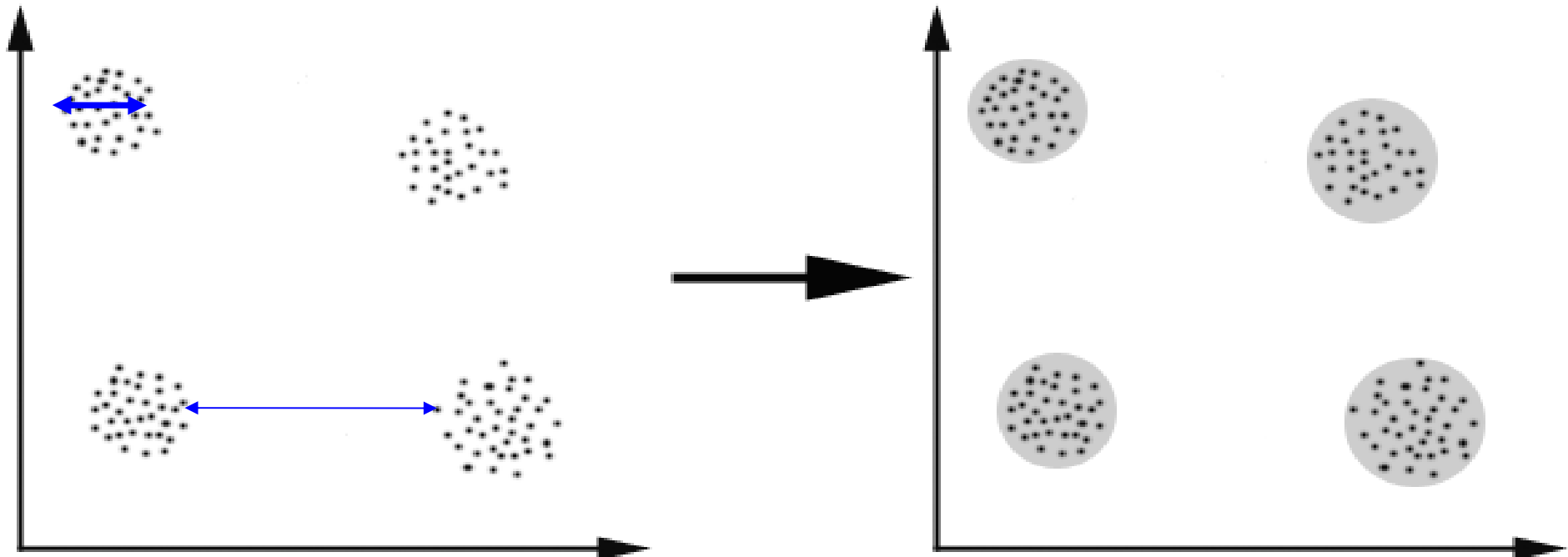
Helburua:

Sail edo *clusterrak* bilatu. Sail bakoitzeko objektuak elkarren ondoan egongo dira eta beste sailetako objektuetik urruti.



Sailkapen-analisia (*cluster analysis*)

Antzekotasuna edo distantzia



Datuak: Erlazioak

Ω , objektuen multzoa

X_j , aldagaiak

x_{ij} , ω_i objektuaren balioa X_j aldagaian

Ω	X_1		X_j		X_p
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ω_i	x_{i1}		x_{ij}		x_{ip}
ω_n	x_{n1}		x_{nj}		x_{np}

p^1	p^2		p^1
p_1^1	p_1^2		p_1^1
p_i^1	p_i^2		p_{i1}
p_n^1	p_n^2		p_n^1

Miatze eta aztertzearen ondorioz emaitzak:

Zenbait erlazio berri

Suitza, 1888: 47 herrialde frankofono

Zenbait adierazle sozioekonomiko (%) eta emankortasuna

X1 Fertility X2 Agriculture X3 Examination
X4 Education X5 Catholic X6 Infant.Mortality

C Clustering

	X1	X2	X3	X4	X5	X6	C		X1	X2	X3	X4	X5	X6	C
Courtelary	80.2	17.0	15	12	9.96	22.2	?	Oron	72.5	71.2	12	1	2.40	21.0	?
Delemont	83.1	45.1	6	9	84.84	22.2	?	Payerne	74.2	58.1	14	8	5.23	23.8	?
Franches-Mnt	92.5	39.7	5	5	93.40	20.2	?	Paysd'enhaut	72.0	63.5	6	3	2.56	18.0	?
Moutier	85.8	36.5	12	7	33.77	20.3	?	Rolle	60.5	60.8	16	10	7.72	16.3	?
Neuveville	76.9	43.5	17	15	5.16	20.6	?	Vevey	58.3	26.8	25	19	18.46	20.9	?
Porrentruy	76.1	35.3	9	7	90.57	26.6	?	Yverdon	65.4	49.5	15	8	6.10	22.5	?
Broye	83.8	70.2	16	7	92.85	23.6	?	Conthey	75.5	85.9	3	2	99.71	15.1	?
Glane	92.4	67.8	14	8	97.16	24.9	?	Entremont	69.3	84.9	7	6	99.68	19.8	?
Gruyere	82.4	53.3	12	7	97.67	21.0	?	Herens	77.3	89.7	5	2	100.00	18.3	?
Sarine	82.9	45.2	16	13	91.38	24.4	?	Martigwy	70.5	78.2	12	6	98.96	19.4	?
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La Vallee	54.3	15.2	31	20	2.15	10.8	?	Val de Ruz	77.6	37.6	15	7	4.97	20.0	?
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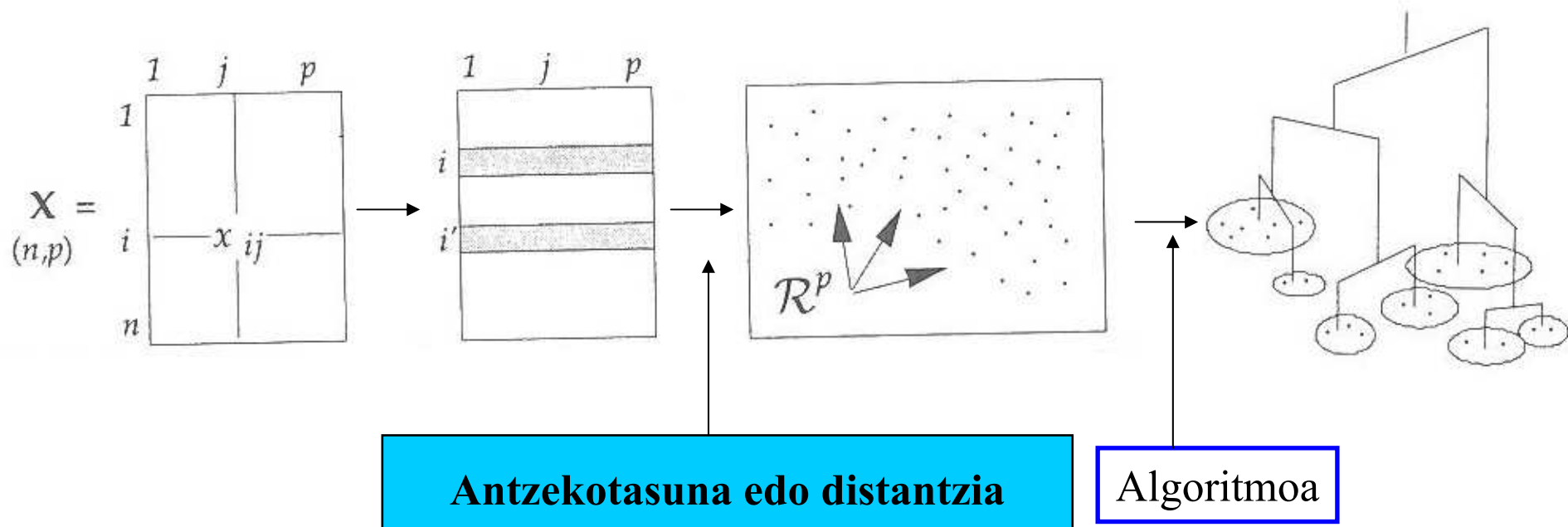
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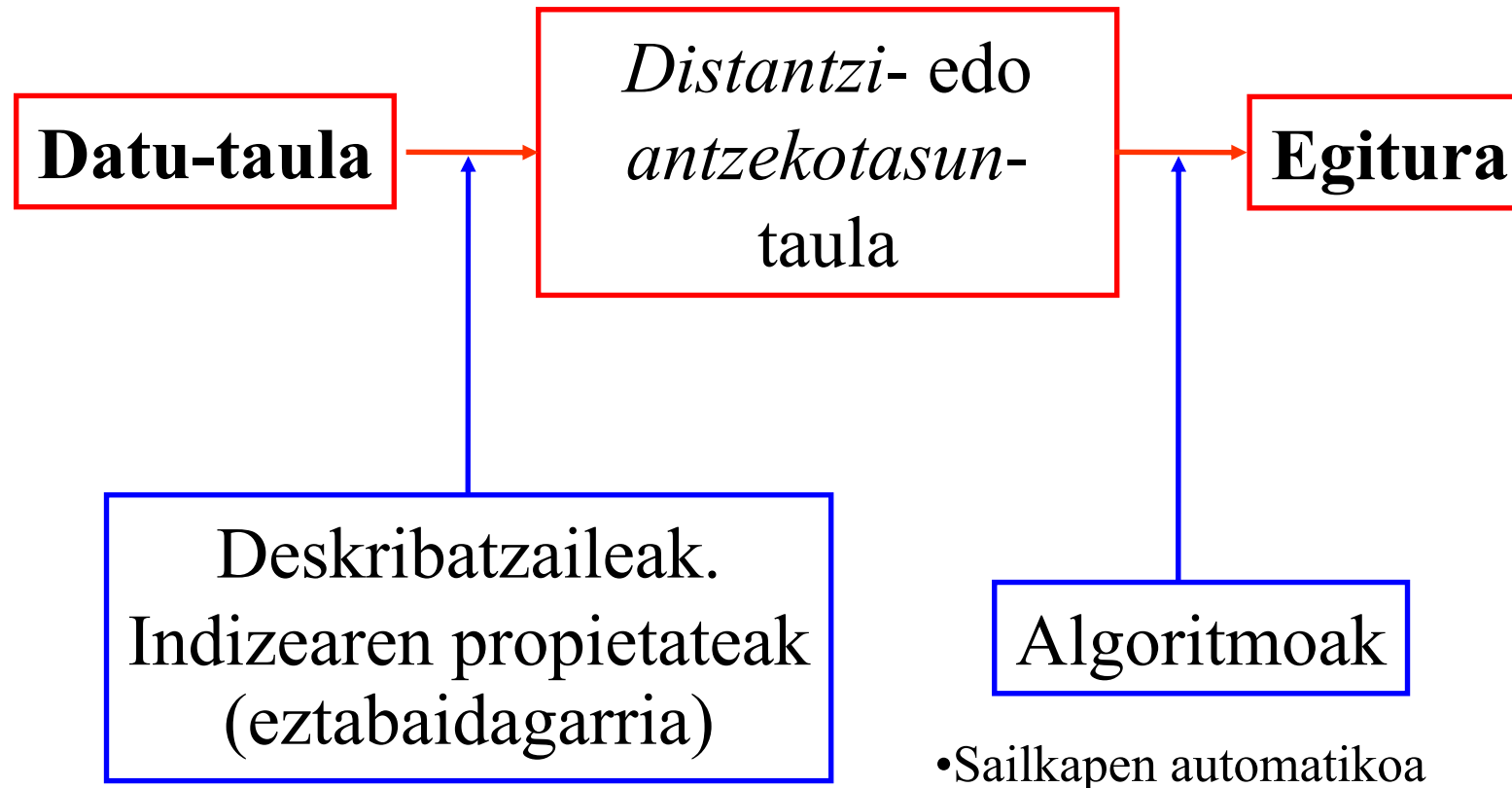
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Grandson	71.7	34.0	17	8	3.30	20.0	Le Locle	72.7	16.7	22	13	11.22	18.9
Lausanne	55.7	19.4	26	28	12.11	20.2	Neuchatel	64.4	17.6	35	32	16.92	23.0
La Vallee	54.3	15.2	31	20	2.15	10.8	Val de Ruz	77.6	37.6	15	7	4.97	20.0
Lavaux	65.1	73.0	19	9	2.84	20.0	ValdeTravers	67.6	18.7	25	7	8.65	19.5
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Sailkapen-analisia (*cluster analysis*)



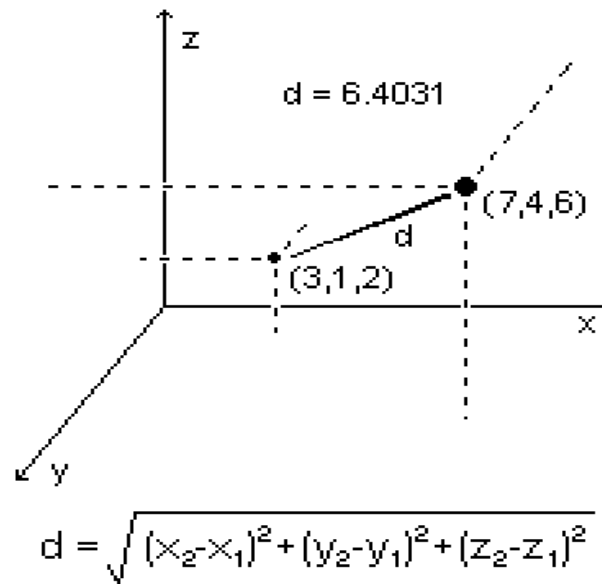
Analisi baten emaitzak hauen menpe daude:

- Eskuratutako datu-multzoan eta beren prozesamenduan.
- Objektuak beren artean alderatzeko erabili den distantzia edo antzekotasun-neurrian.
- Erabilitako algoritmoan.



Distantzia

Datuak errepresentatzen dituzten bektoreen osagaiak unitate berberaz baldin badaude adierazita (aldagai homogeneoak), *distantzia euklidentarra* nahikoa izan daiteke sailkatze-lanak egiteko, bestela (aldagai heterogeneoak) ez.



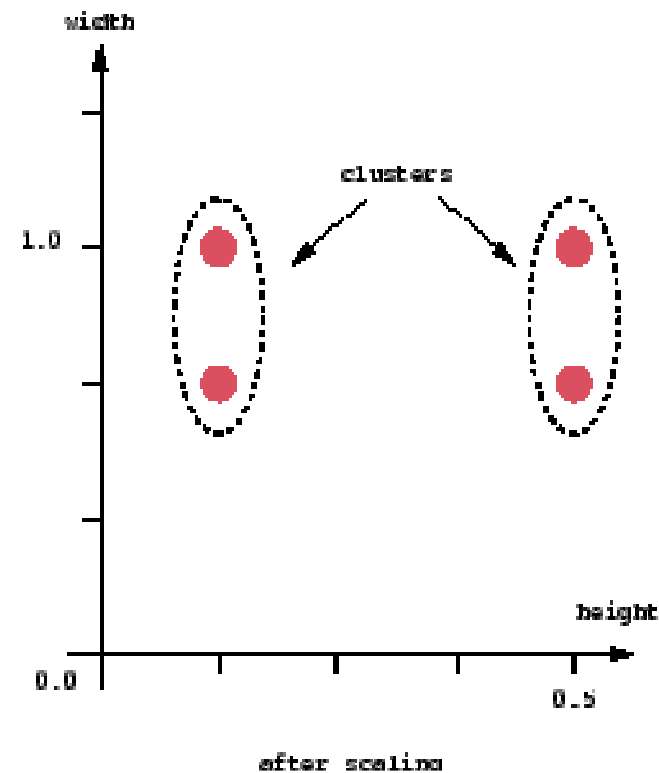
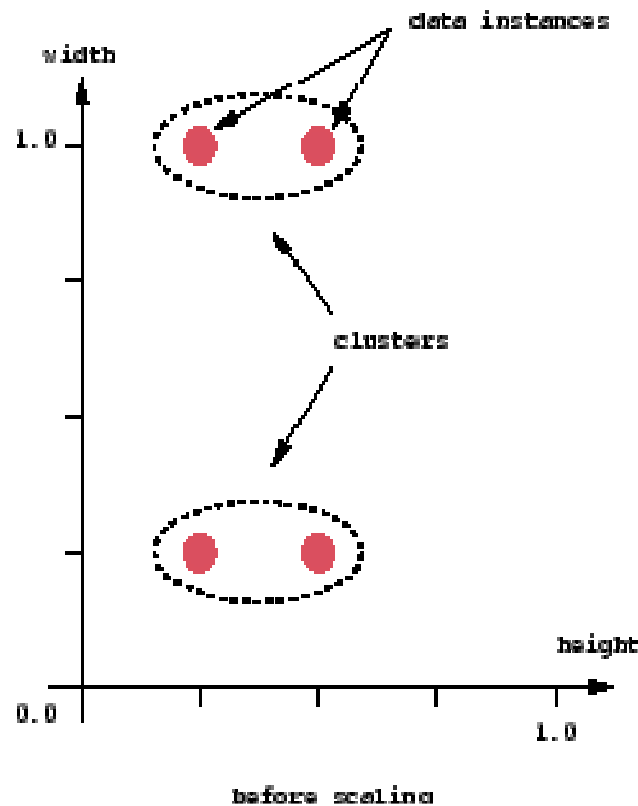
$$P = (p_1, p_2, \dots, p_n) \quad Q = (q_1, q_2, \dots, q_n).$$

$$\sqrt{(p_1 - q_1)^2 + (p_2 - q_2)^2 + \dots + (p_n - q_n)^2} = \sqrt{\sum_{i=1}^n (p_i - q_i)^2}$$

Distantzia

Aldagaiak heterogeneoak badira ere, batzutan ez da komenigarri izaten distantzia euklidestarra zuzenean erabiltzea.

Aldagaien eskala ezberdinak sailkatze ezberdinak eman dezakete.



Distantzia

Arazoa neurri-bektore baten osagaiak formula batean konbinatzeko eran datza, gero *clustering*eko algoritmo batean erabilia izan dadin: nola aukeratu distantzia?

Erantzuna arazoaren arloaren ezagutzan aurkitzen da.

Distantzia bat aukeratzeko ez dago gida teorikorik.

Minkowski-ren distantzia orokorra: _

$$d_p(x_i, x_j) = \left(\sum_{k=1}^d |x_{i,k} - x_{j,k}|^p \right)^{\frac{1}{p}}$$

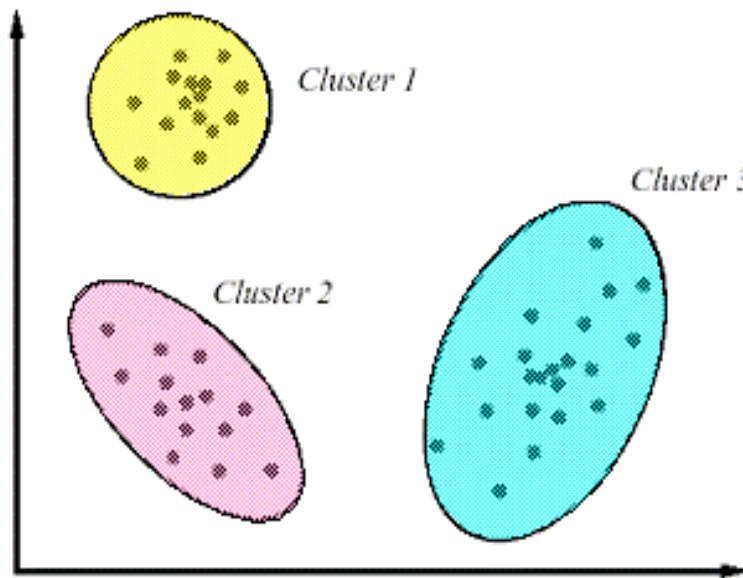
$p=2$, *Distantzia euklidestarra*.

$p=1$, *Hiriko blokeen distantzia (Manhattan distantzia)*.

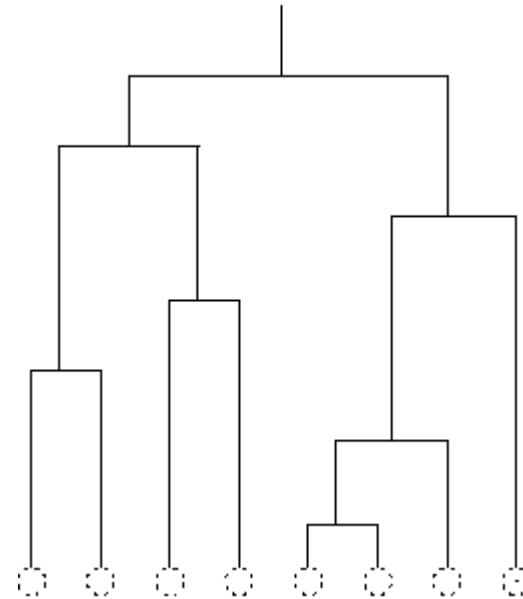
Algoritmoak

Clusteringeko algoritmo gehienak bitan bana daitezke, ematen duten *egituraren* arabera:

Partiketarako algoritmoak



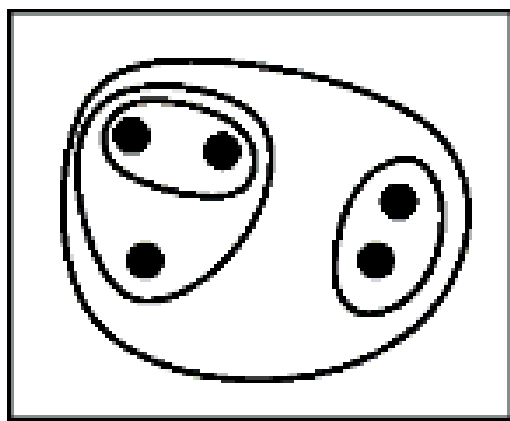
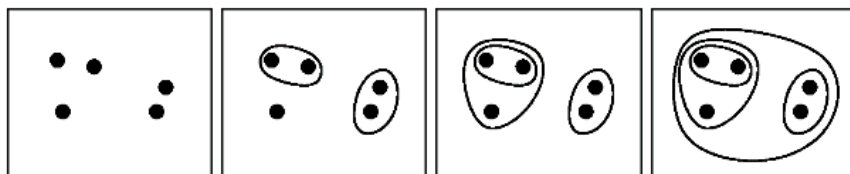
Hierarkiarako algoritmoak



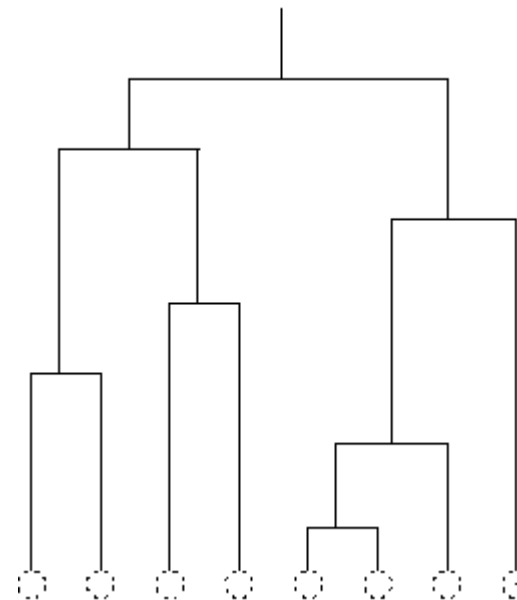
Algoritmoak

Hierarkiarako errepresentazioak:

Venn-en diagramak

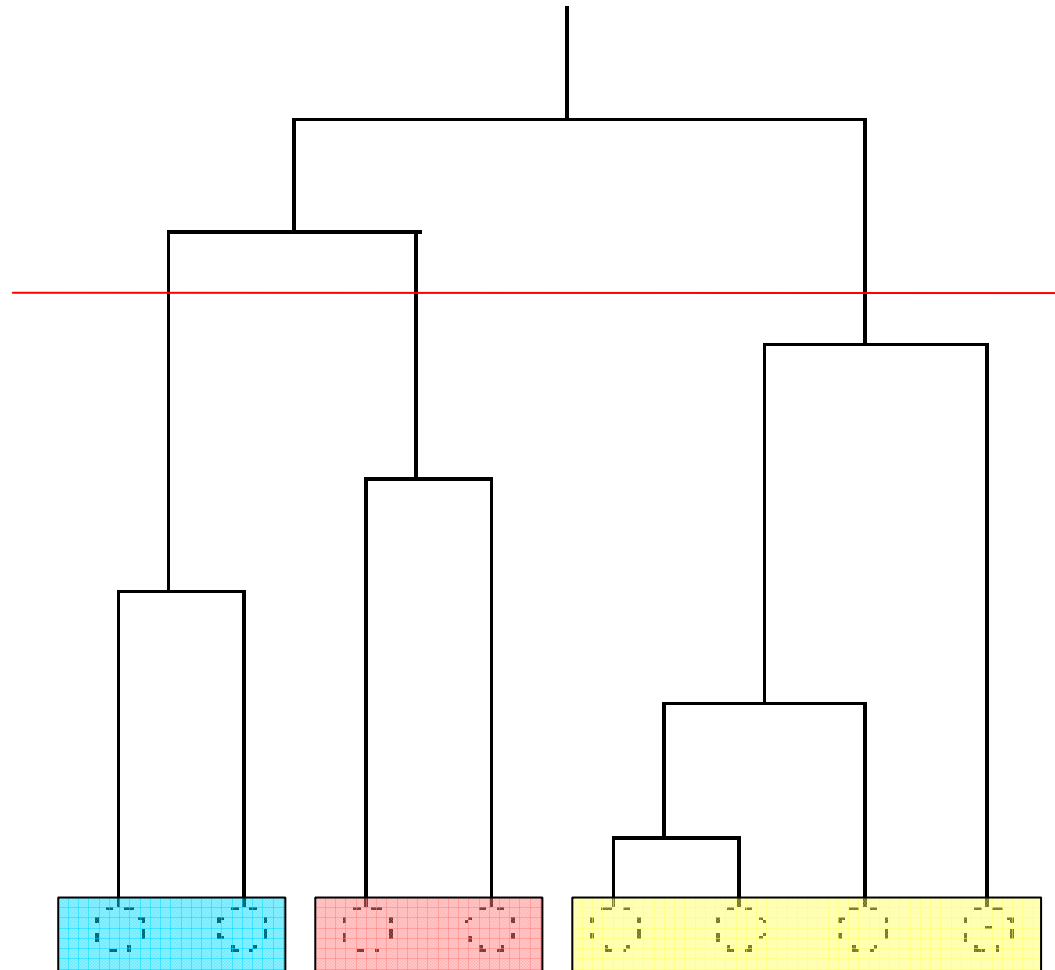


Dendrograma



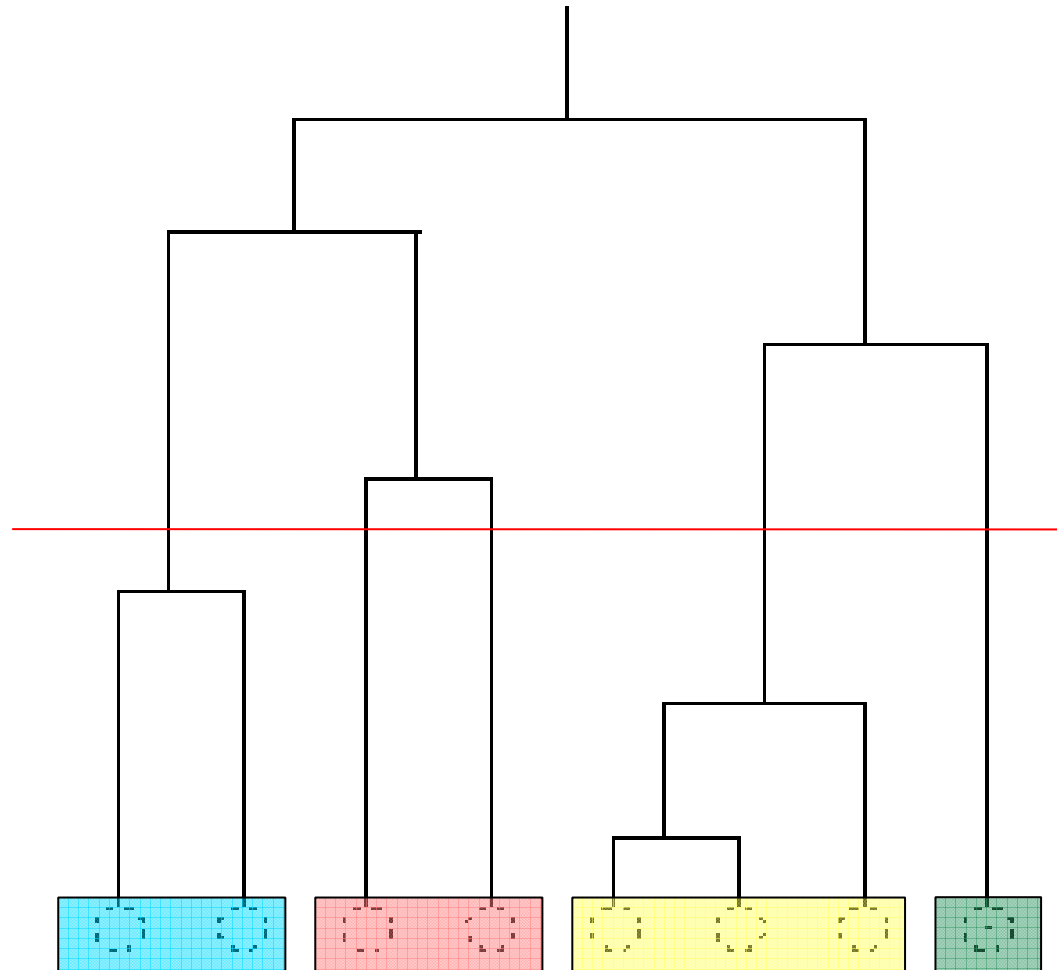
Algoritmoak

Hierarkia-rako algoritmoek partiketa-sorta bat ematen dute:



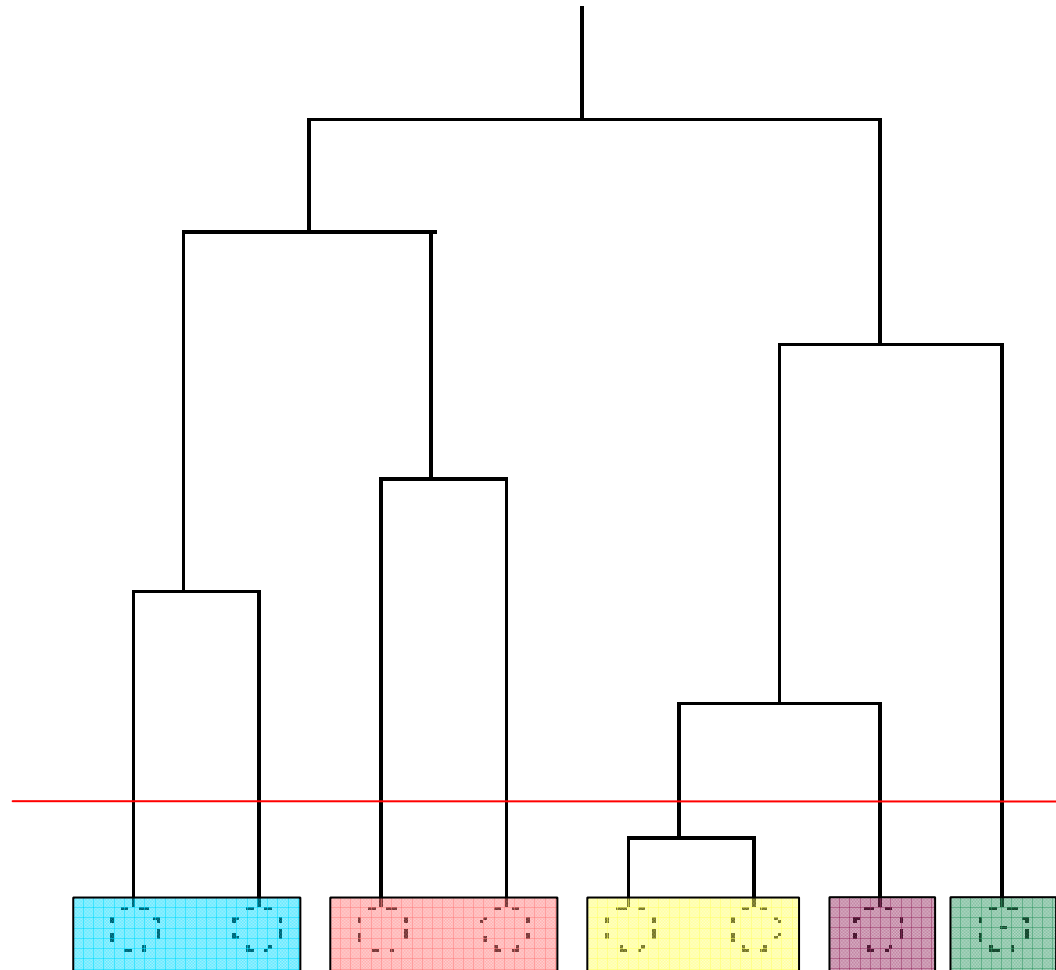
Algoritmoak

Hierarkia-rako algoritmoek partiketa-sorta bat ematen dute:



Algoritmoak

Hierarkia-rako algoritmoek *partiketa*-sorta bat ematen dute:



Partiketarako algoritmoa: k-means

Dimentsio anitzeko espazio batean errepresentatuta dauden n objektu emanez gero, eta eraiki behar diren k sail edo *cluster*-kopurua finkatuta, **helburua** objektuen beren saileko erdigunera (c_i , *means*, *batezbestekoak*) duten distantzia karratuen batura minimizatzea da.

$$\sum_i \sum_{x \in X_i} d^2(x, c_i)$$

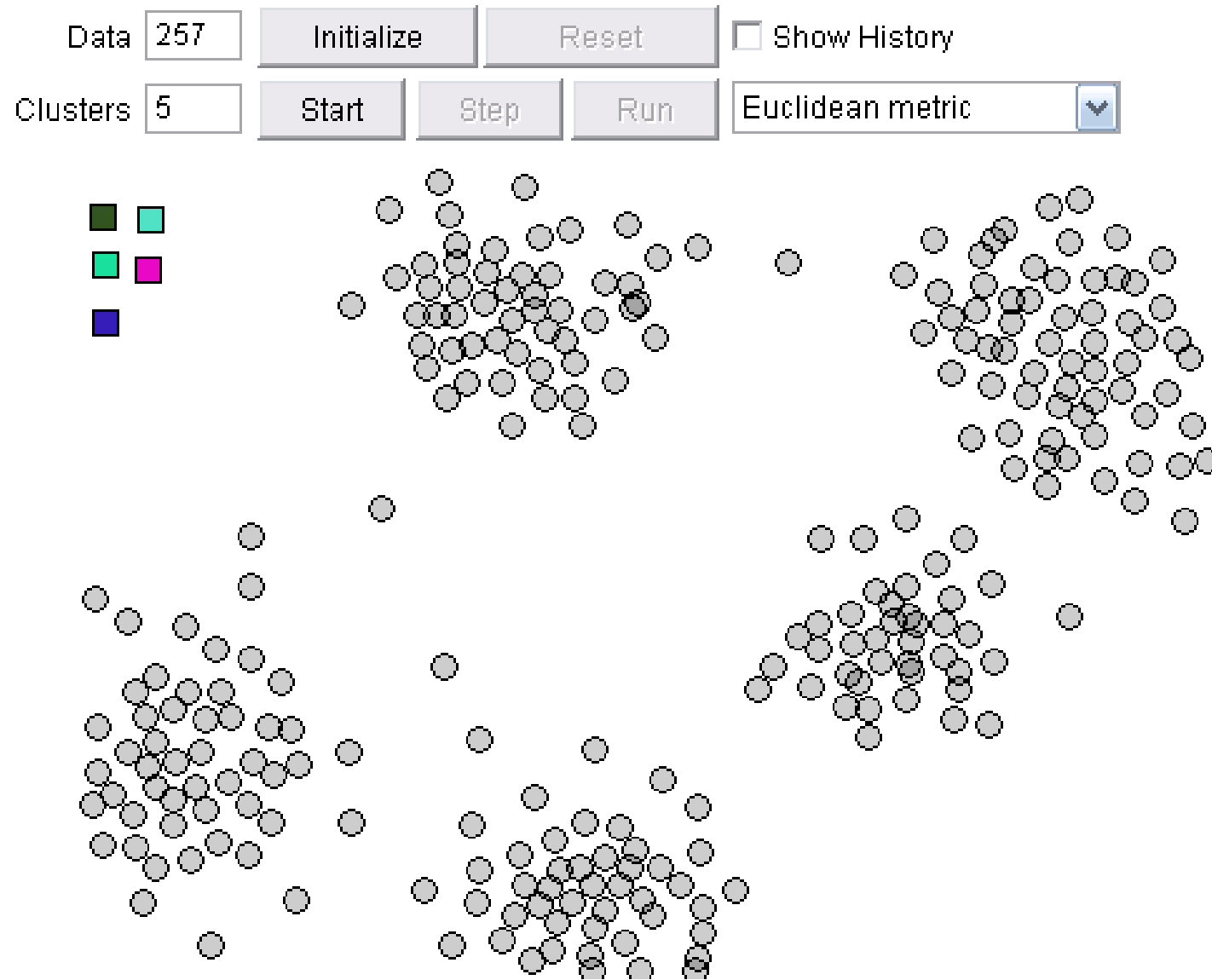
Partiketarako algoritmoa: k-means

1. Finkatu *cluster*-kopurua: k .
2. Aukeratu k *cluster*ren ‘haziak’.
3. Objektu bat gertuen daukan ‘hazia’ren sailekoa egiten da.
4. k *cluster*retan ‘hazi’ berriak lortzen dira (saileko batezbestekoak).
5. 3. eta 4. urratsetan aritu *clusterrak* egonkortu bitartean (‘haziak’ ez dira aldatzen).

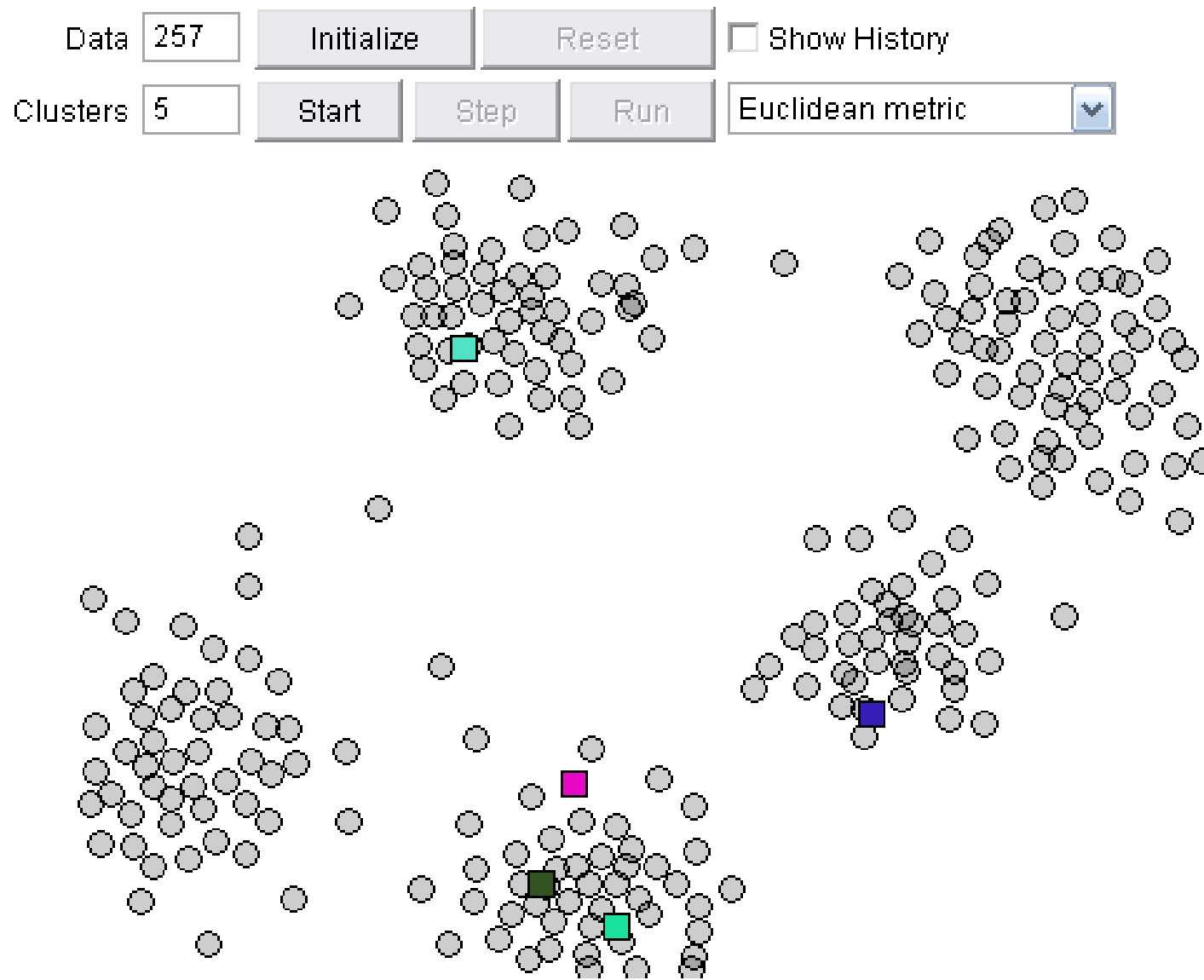
Partiketarako algoritmoa: k-means

- 1. urratsean ez dago argi zein izan behar duen k -ren balioa.
- 2.ean ‘haziak’ zoriz aukeratzen dira objektuen artetik, edo errepresentazio-espazioan, edo, informaziorik badago erabitzailak erabakita.
- 3.ean *distantzia* definitu behar da: *distantzia euklidentarra* da ohikoena.
- 3.ean, objektu guztiak bat-batean esleitu beharrean, gero *means* edo ‘hazi’ berriak kalkulatzearren, pausoz pauso berritu liteke objektu bakoitza *cluster* batean esleitzean.
- 4.ean ‘hazi’aren kontzeptua zabalagoa izan liteke: ‘nukleoa’; puntu-azpimultzo bat, zuzen bat, plano bat, eta abar.

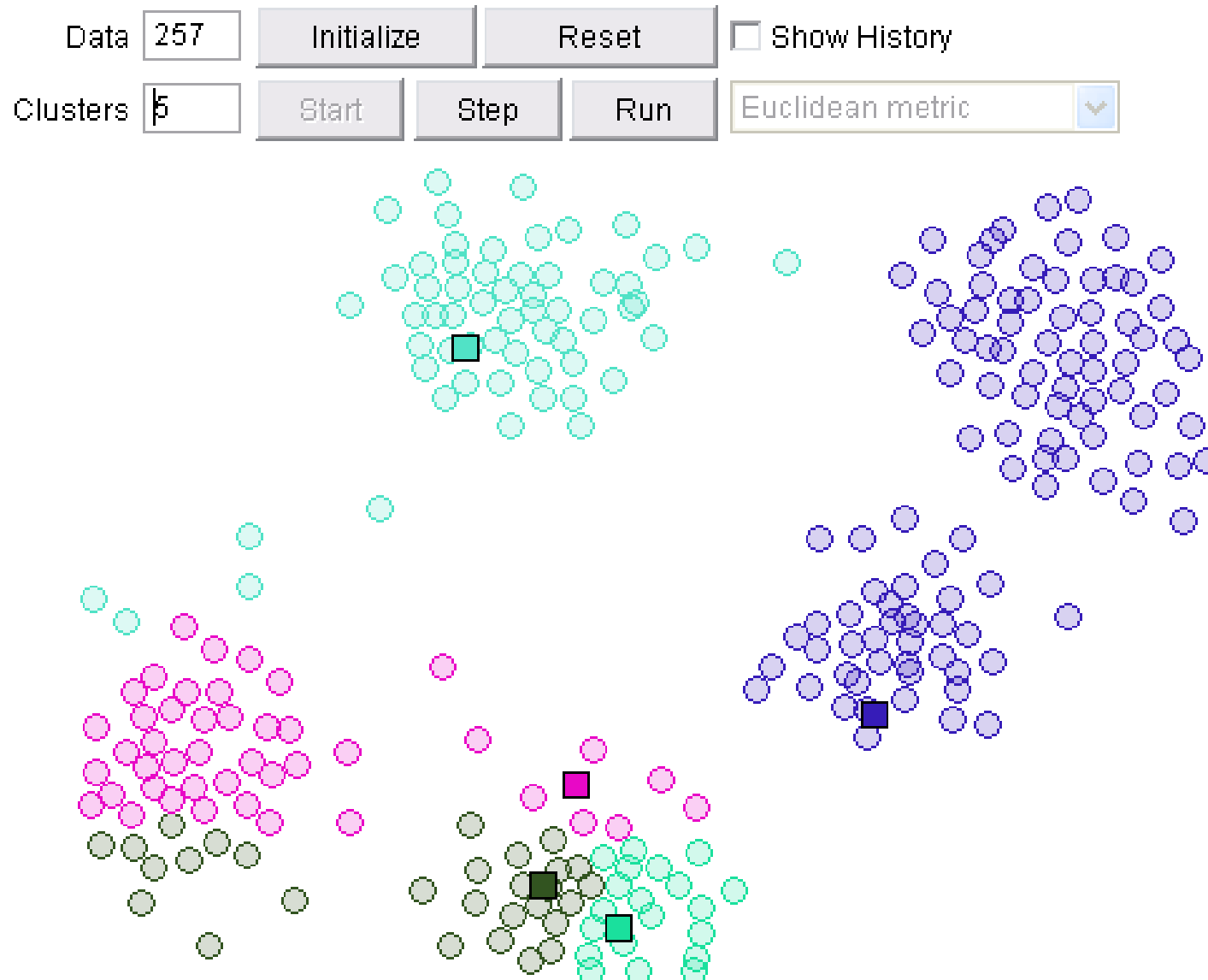
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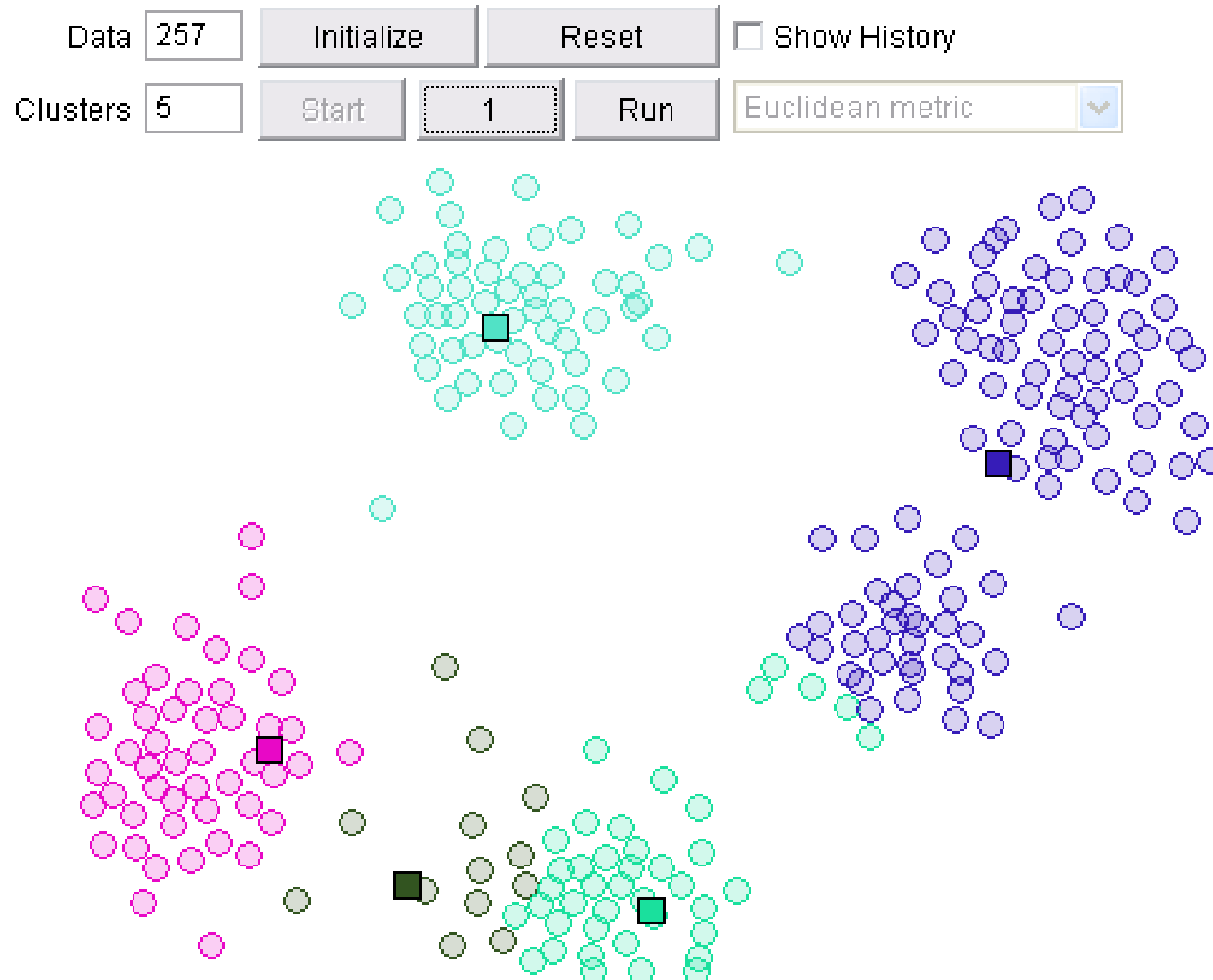
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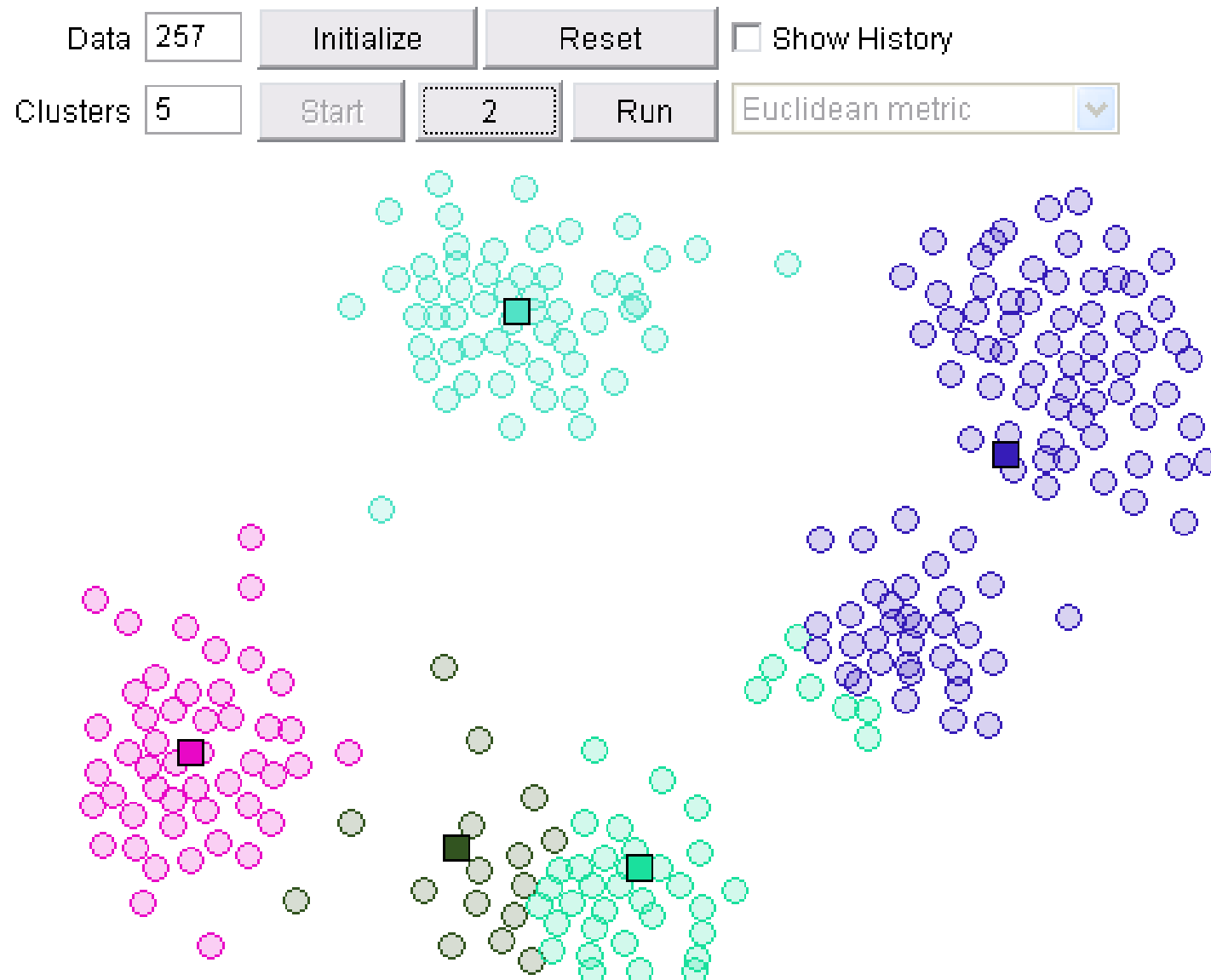
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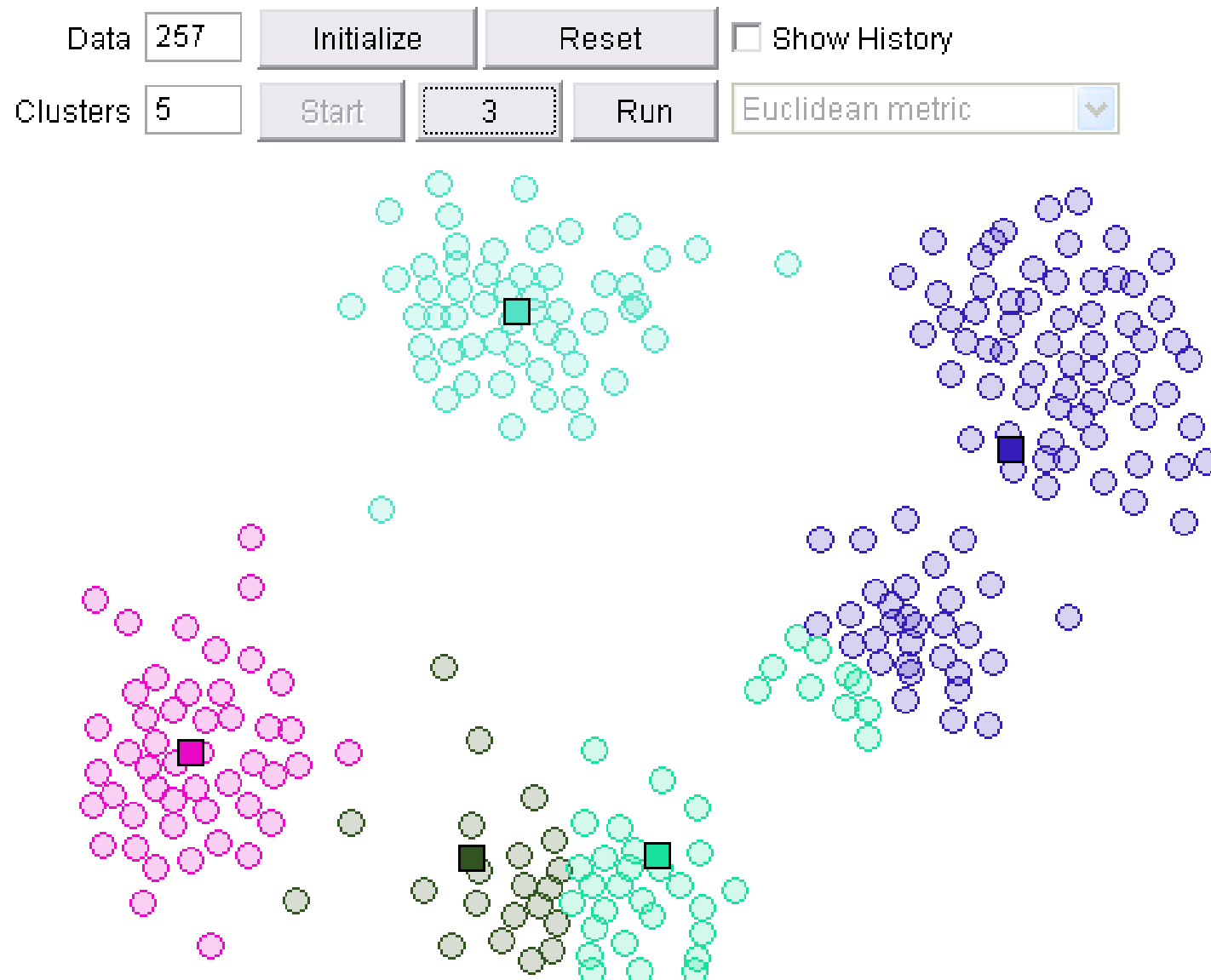
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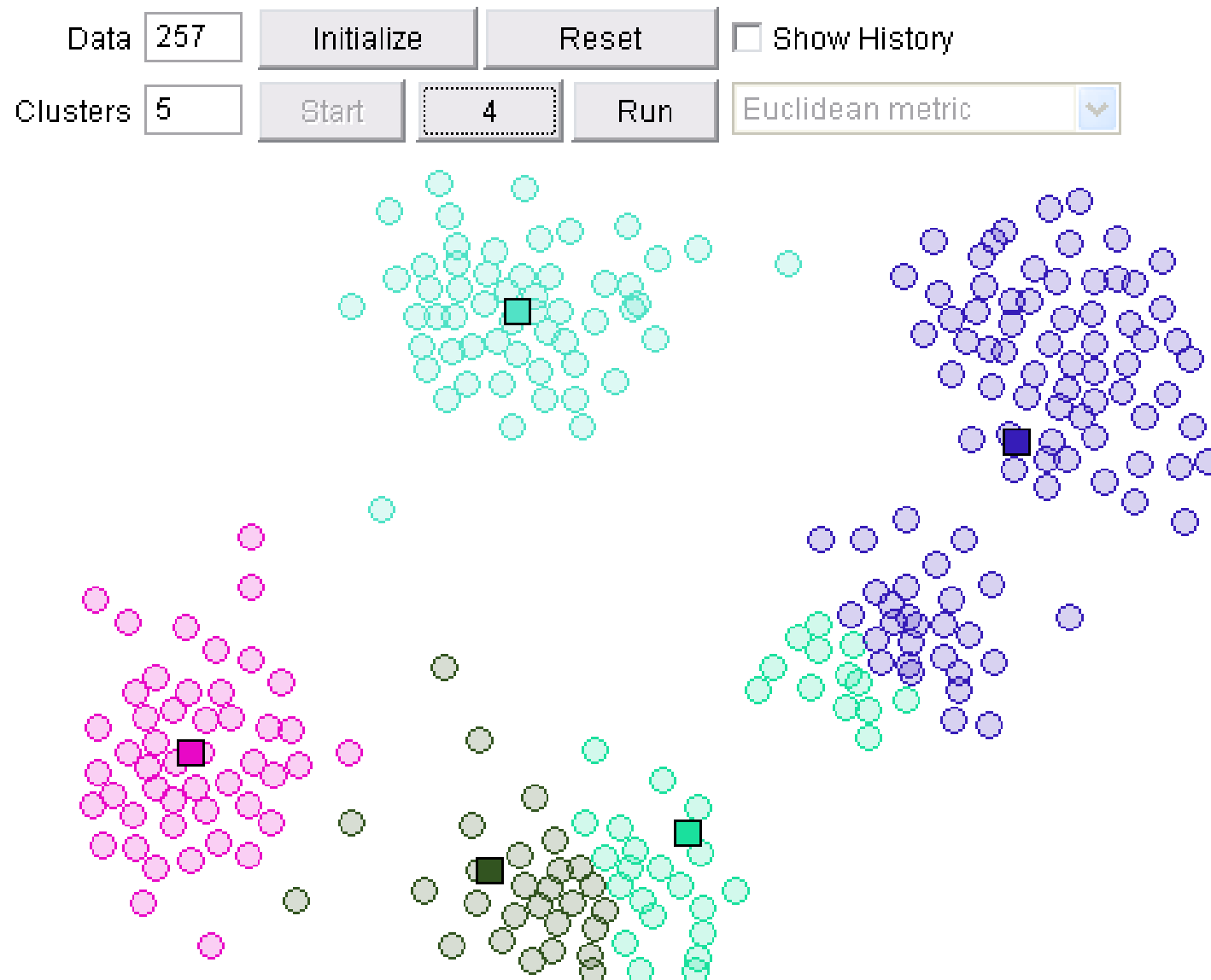
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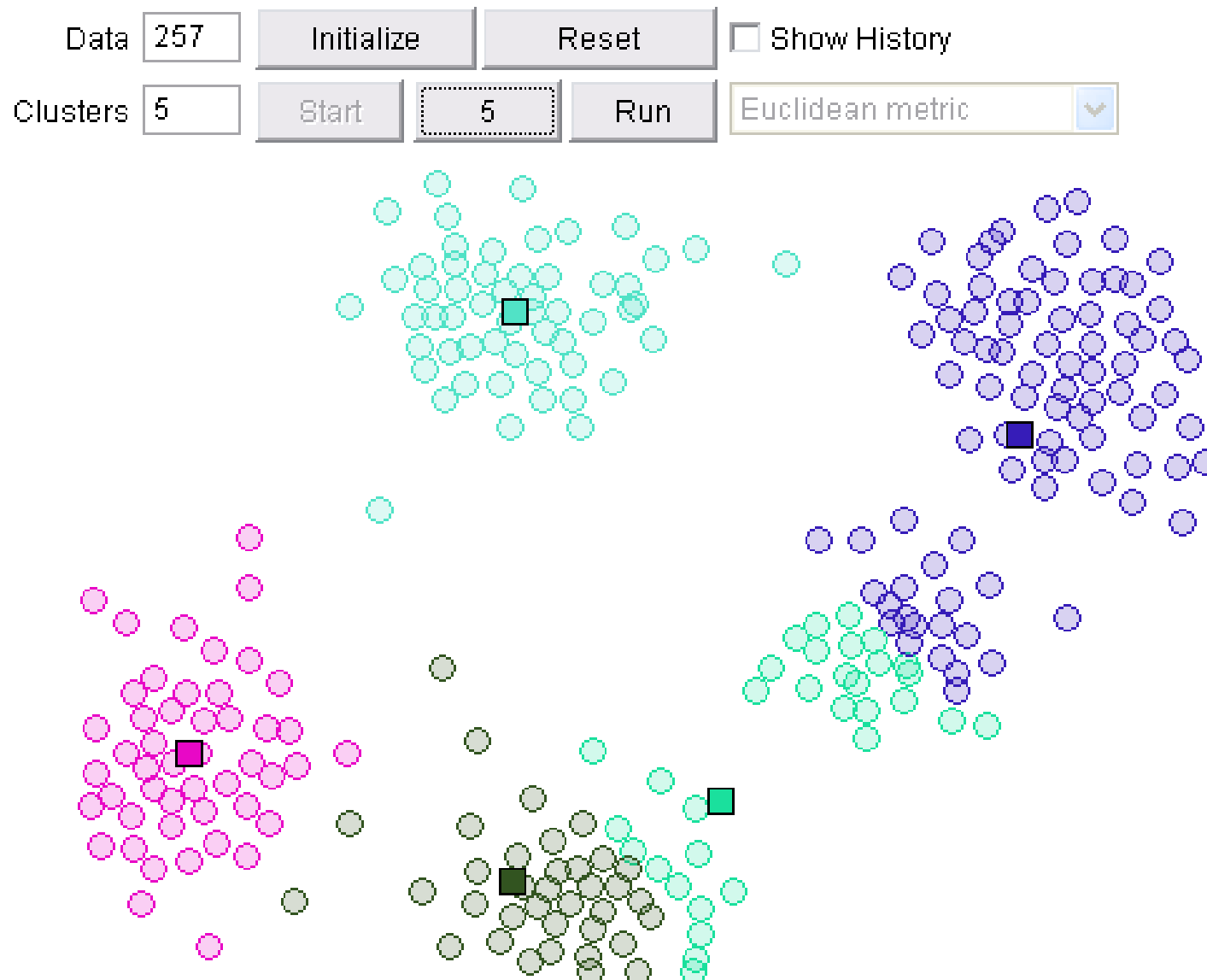
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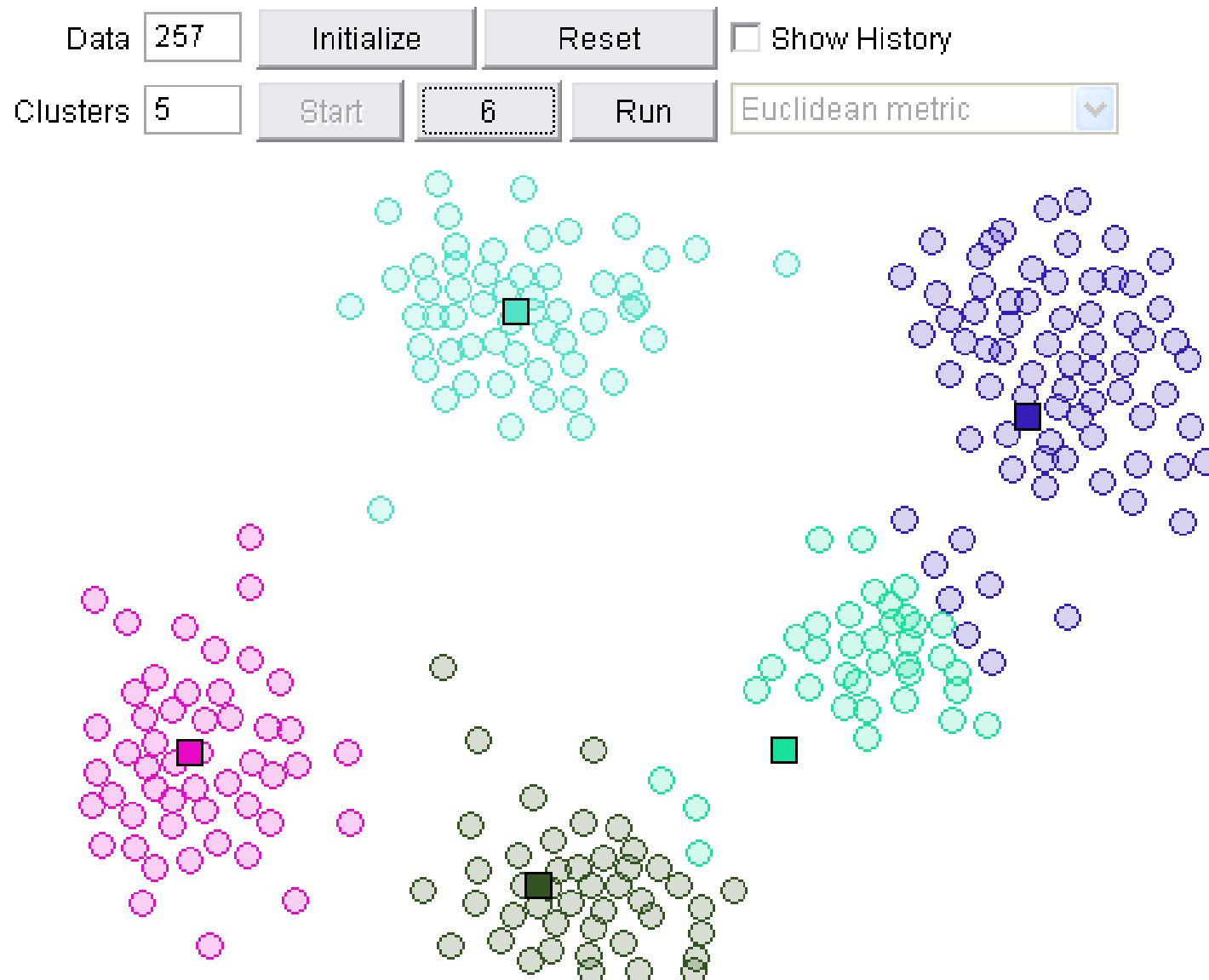
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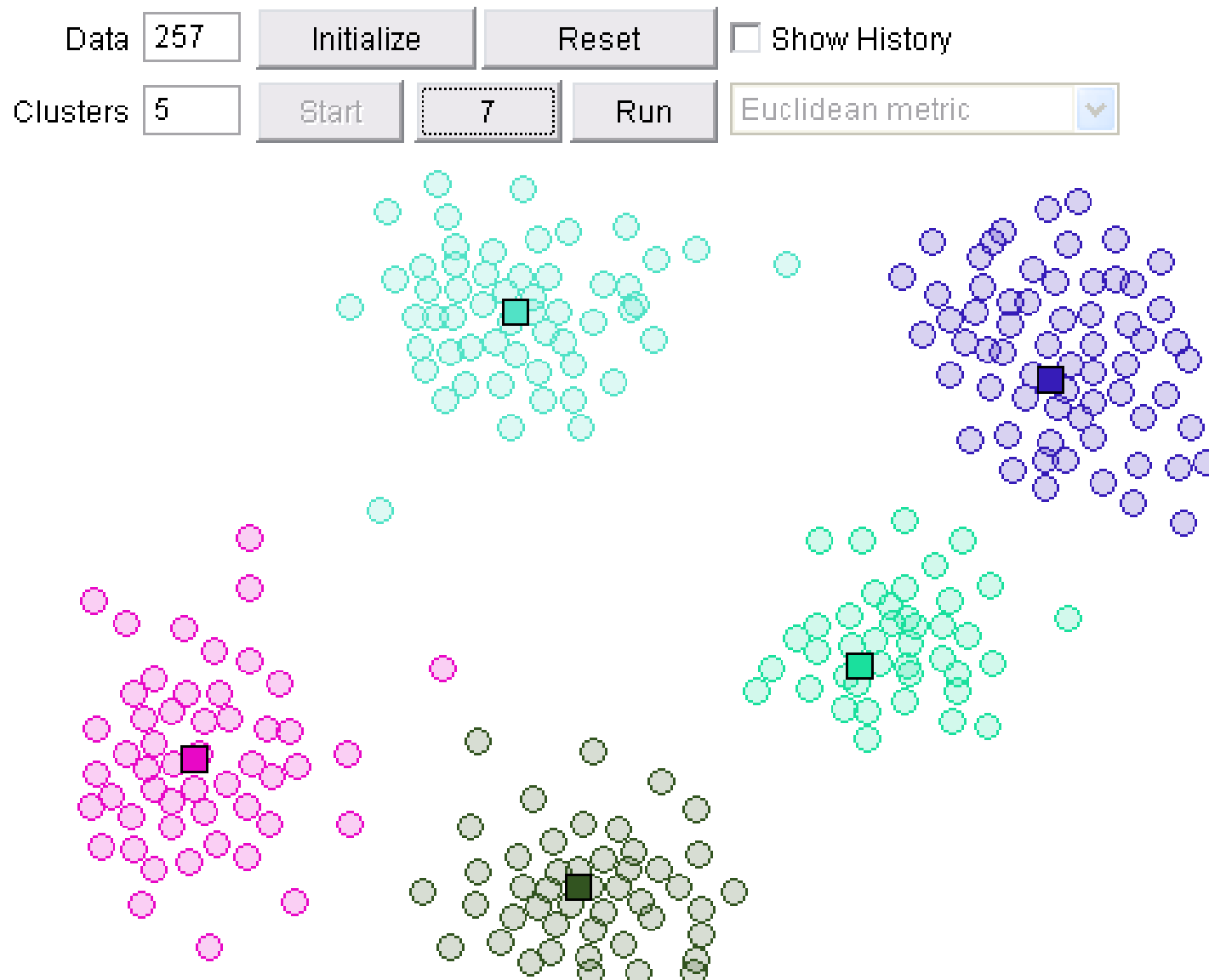
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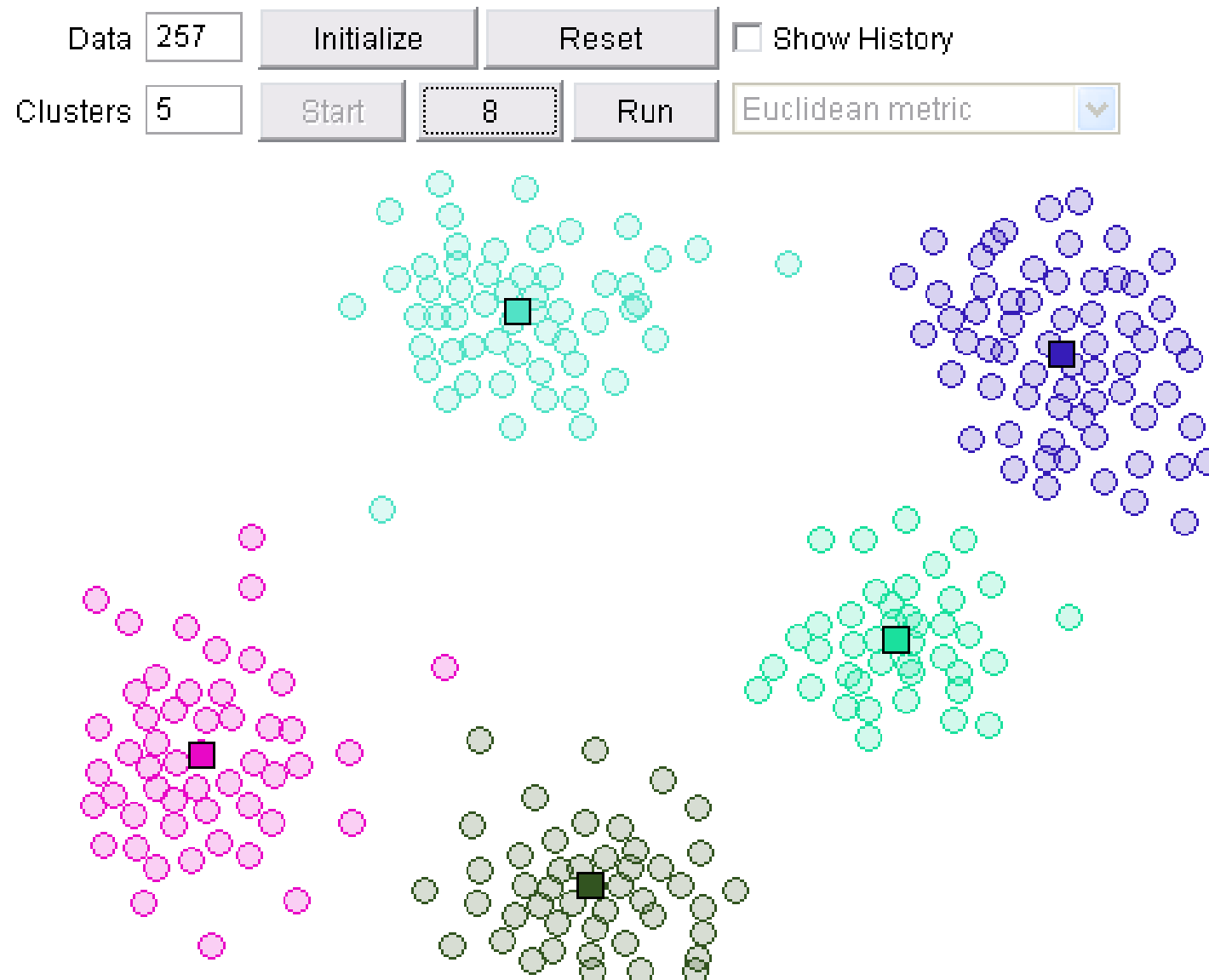
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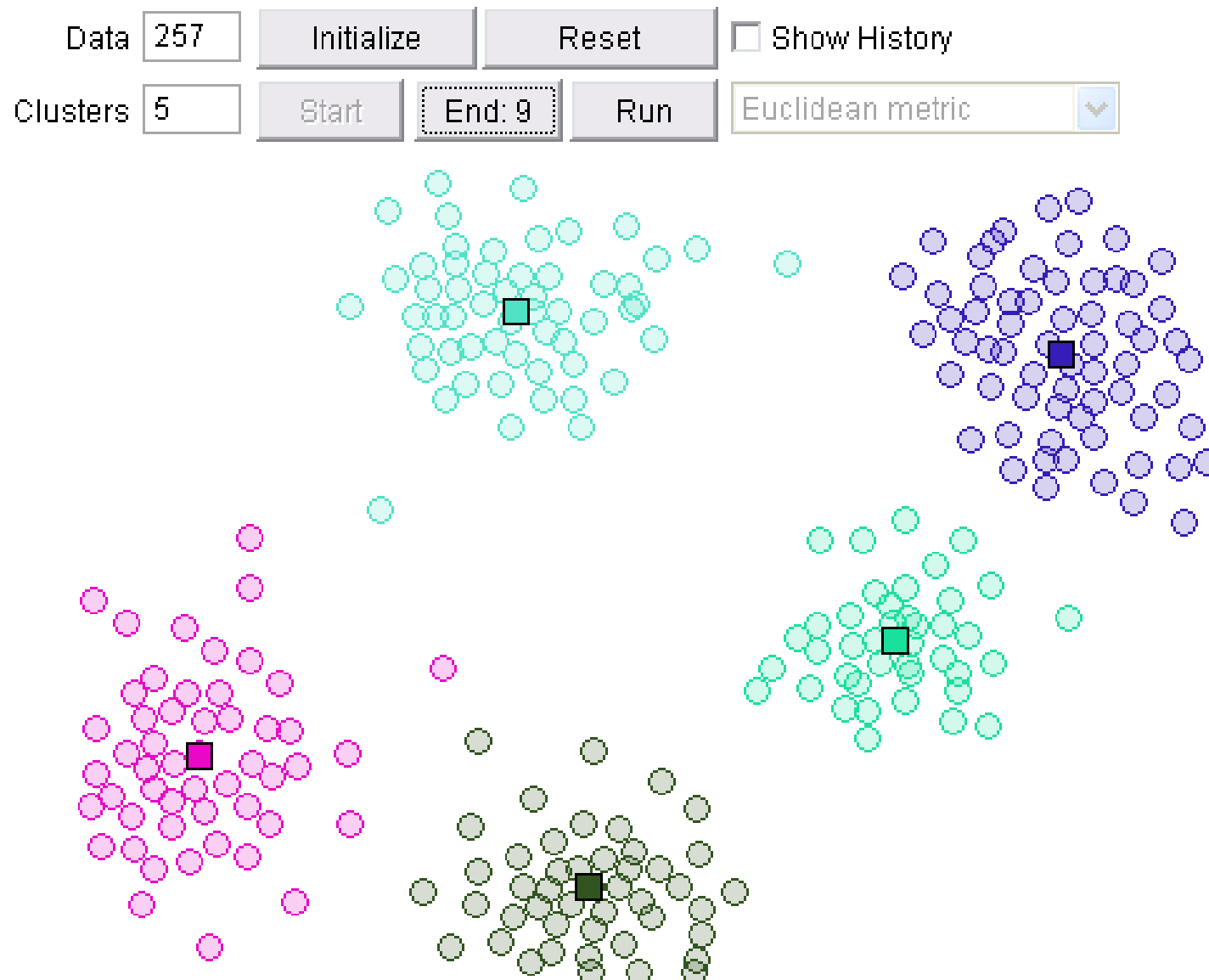
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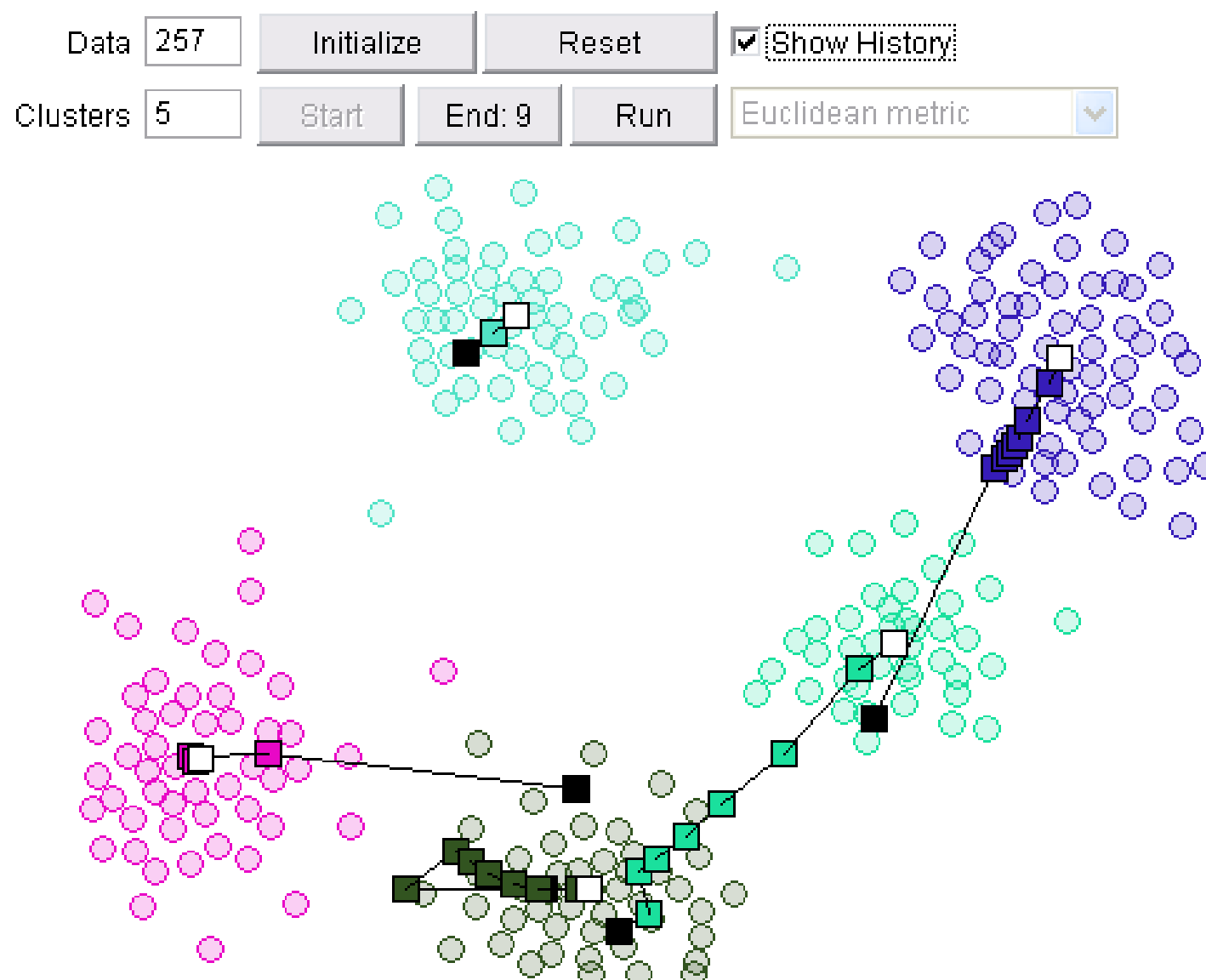
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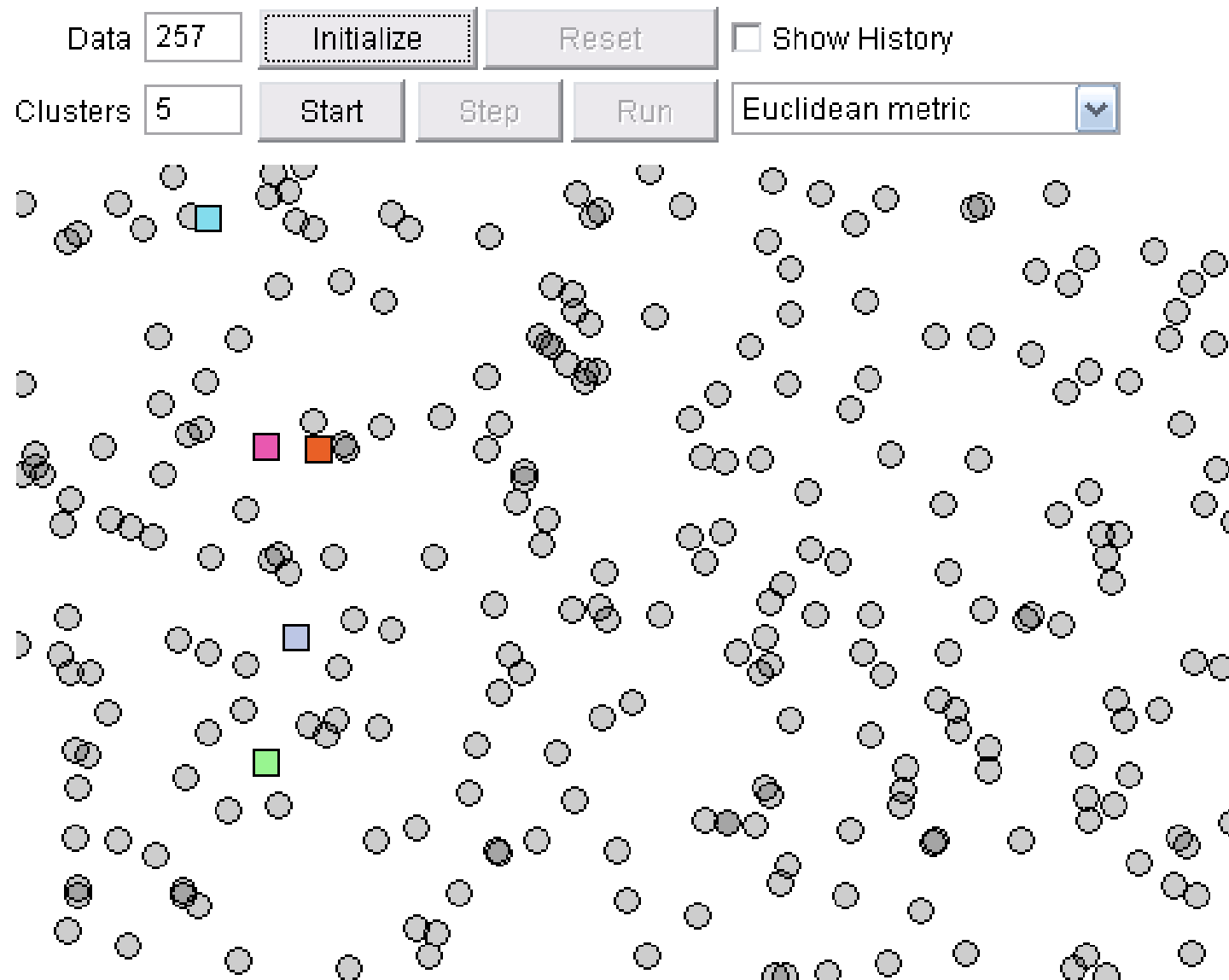
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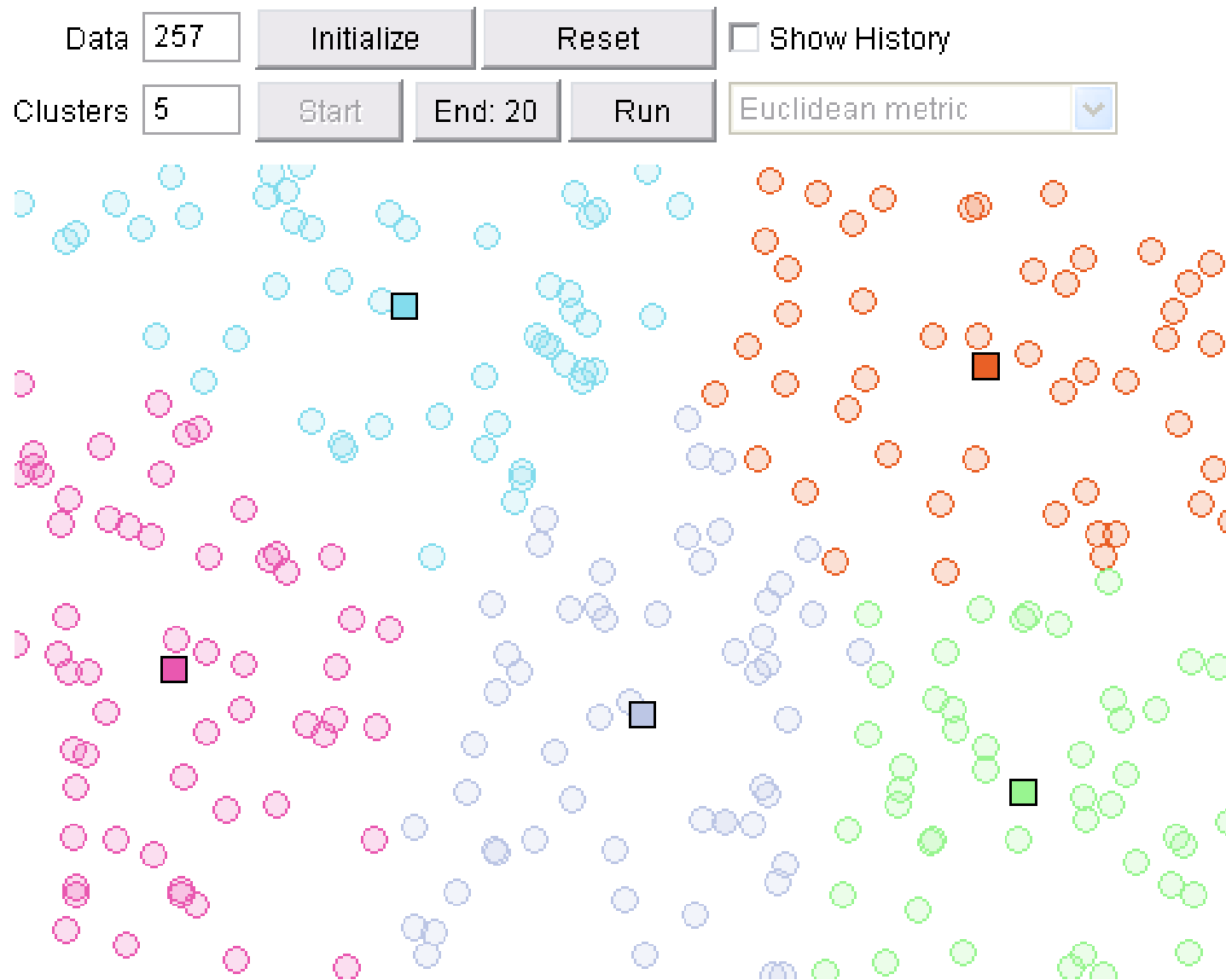
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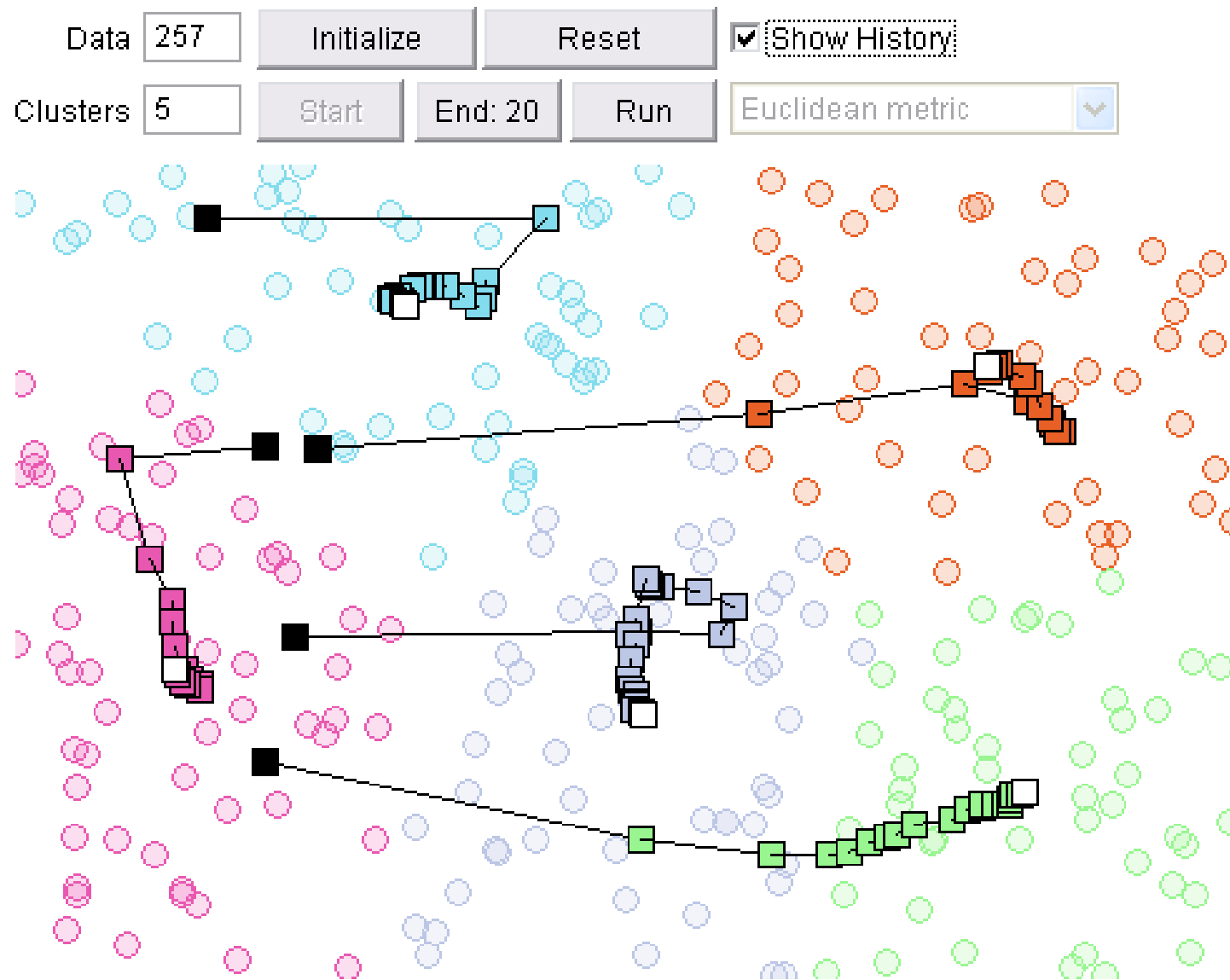
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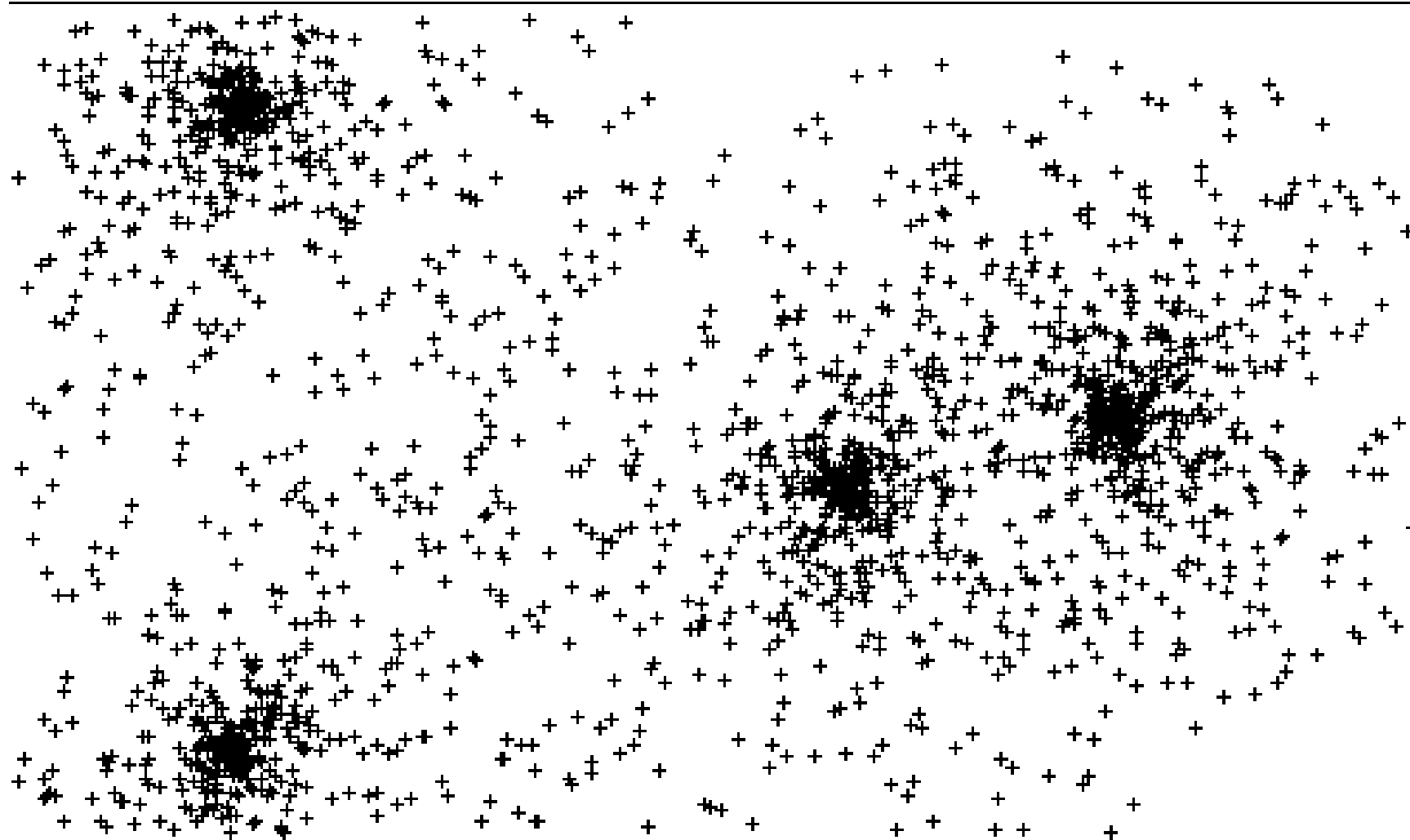


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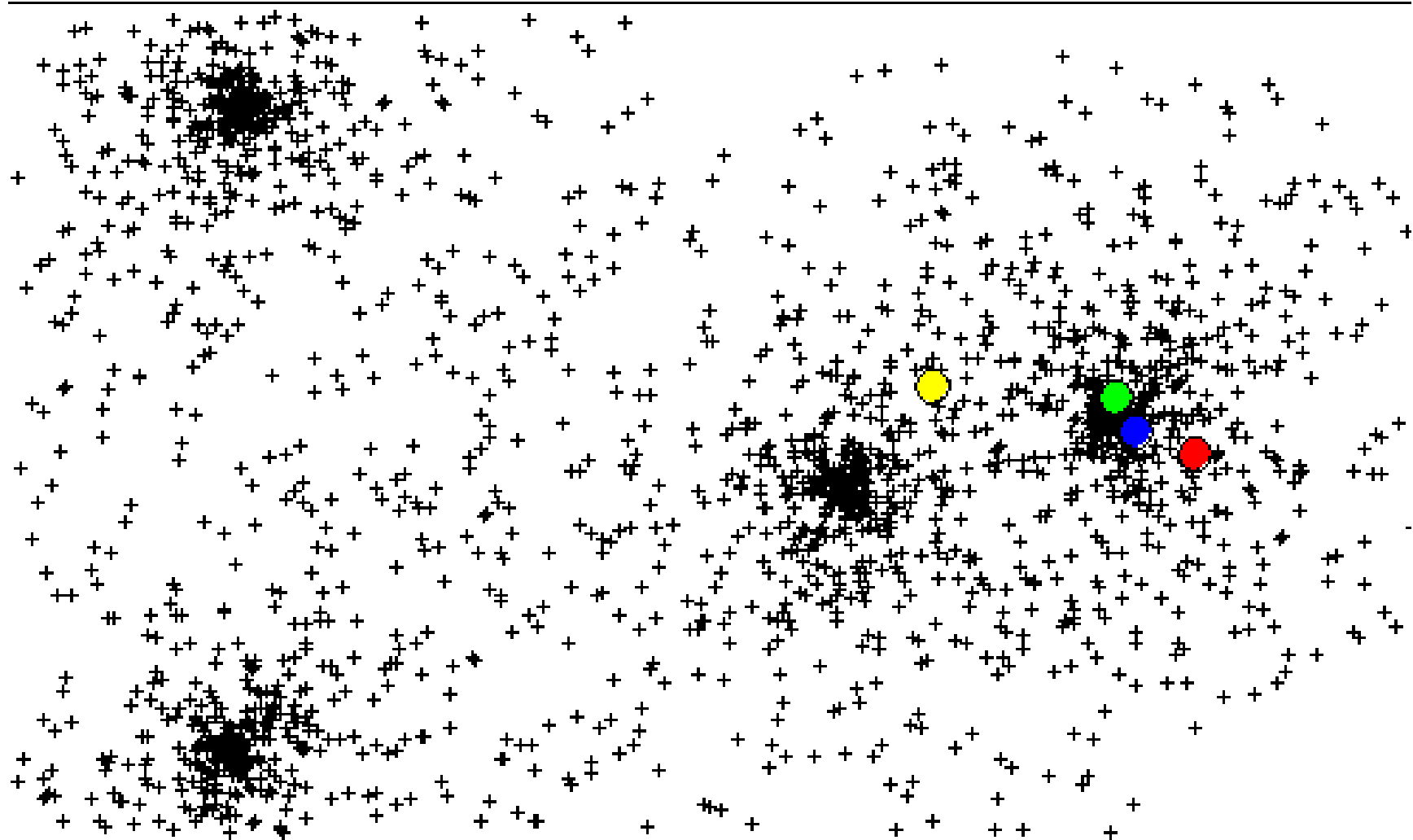
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Start New Start Reset Run Draw Cluster 4 ☐ Show History



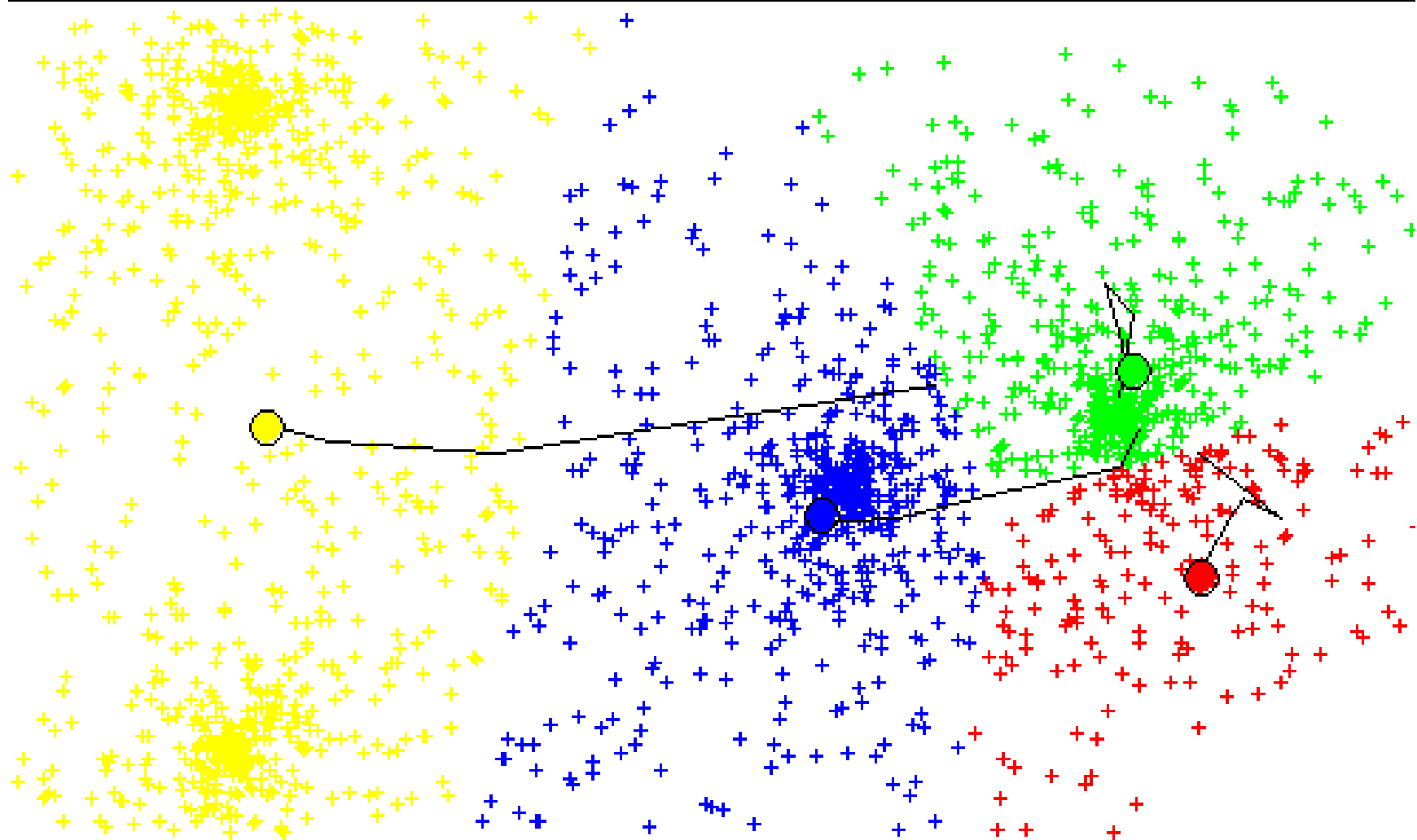
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4 ☒ Show History



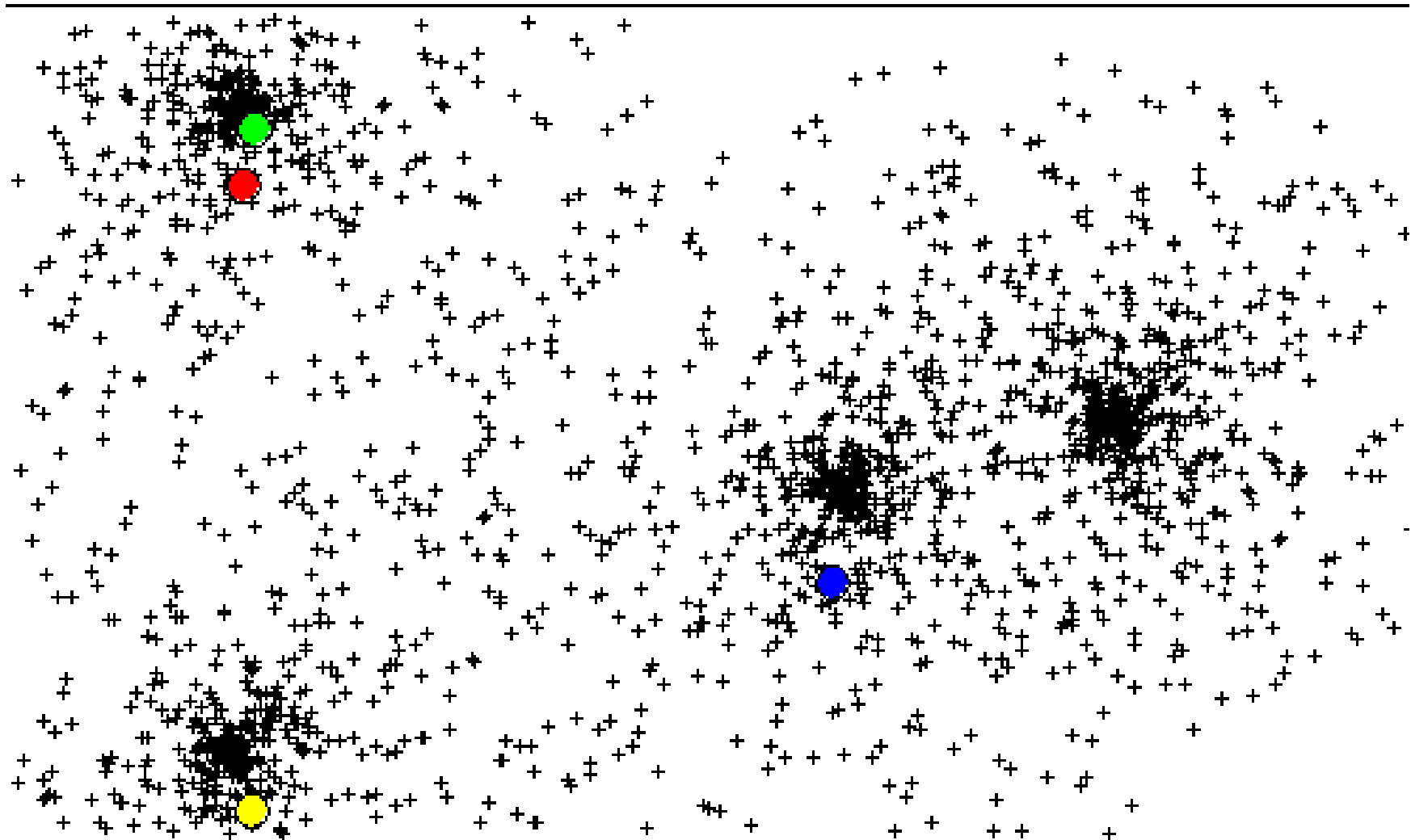
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Step New Start Reset Run Draw Cluster 4 ☒ Show History

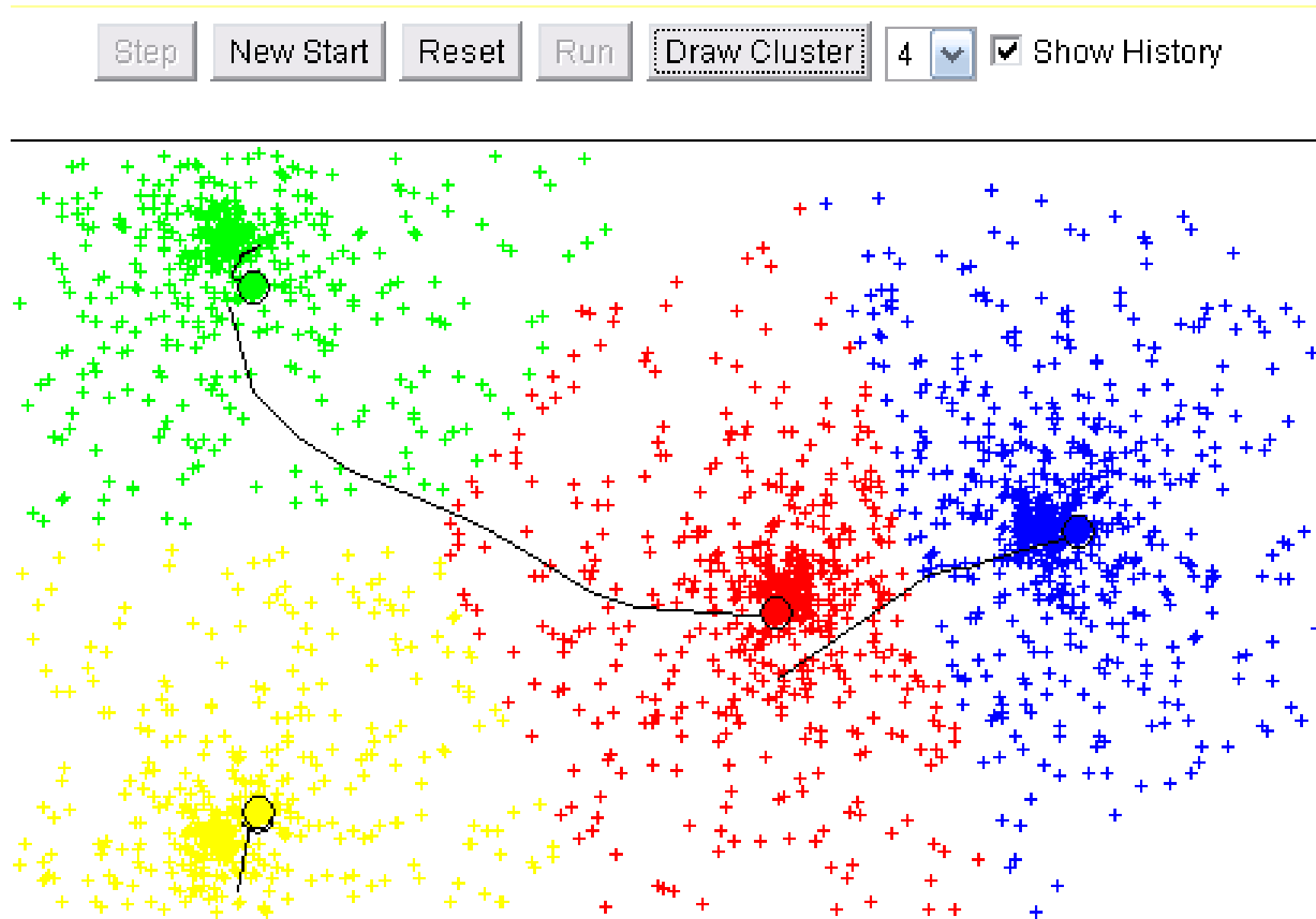


<http://www.rob.cs.tu-bs.de/content/04-teaching/06-interactive/Kmeans/Kmeans.html>

4 ☒ Show History



<http://www.rob.cs.tu-bs.de/content/04-teaching/06-interactive/Kmeans/Kmeans.html>

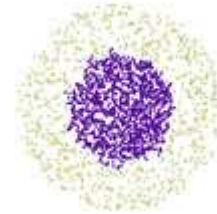
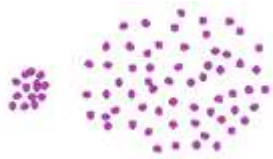


Partiketarako algoritmoa: k-means

Emaitzaren kalitatea, eta beraz, soluzioarena, hasierako hazien menpe dago.

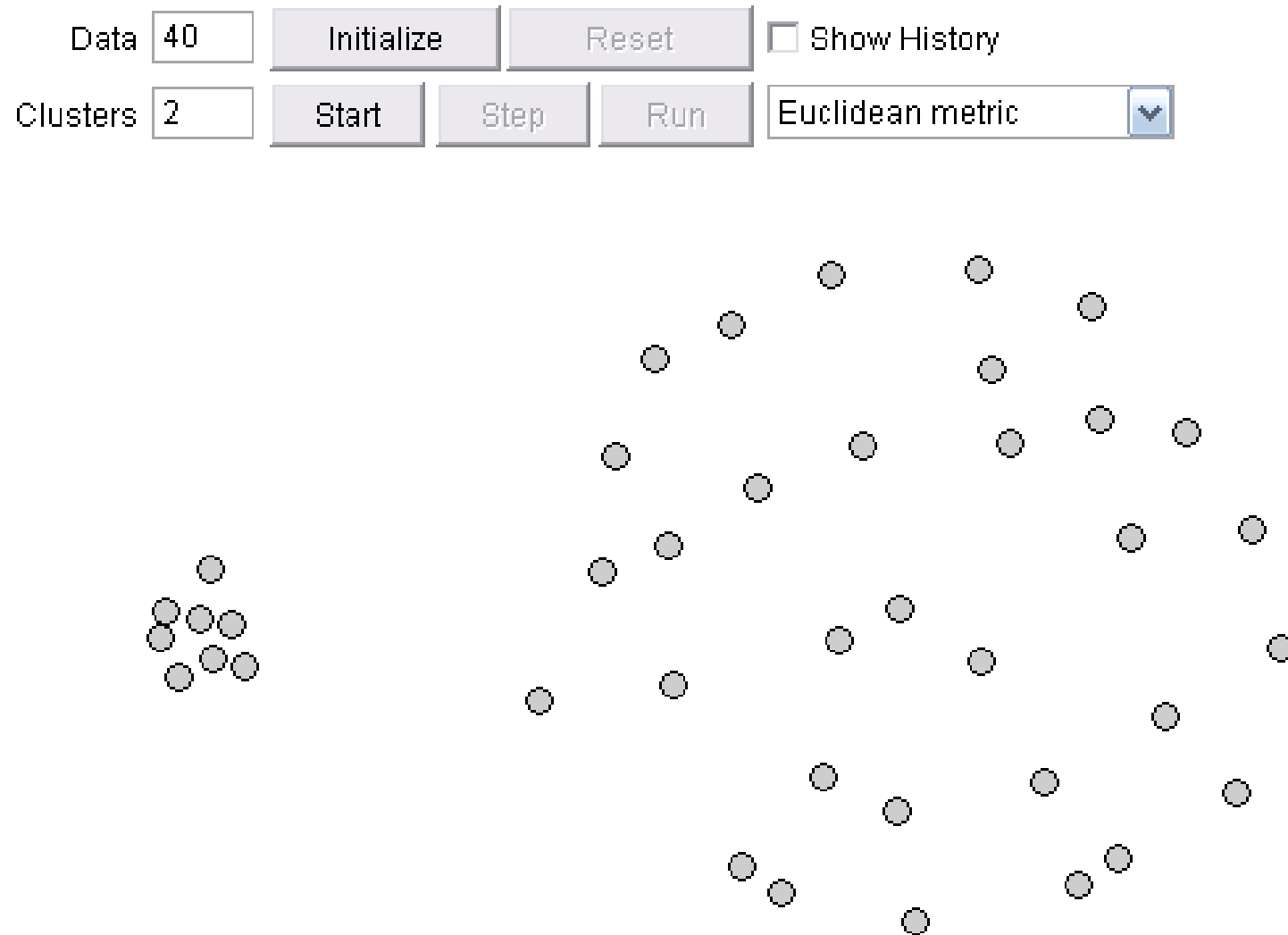
Partiketarako algoritmoa: k-means

Irudi hauetan 2 sail edo cluster ikusten dugu.

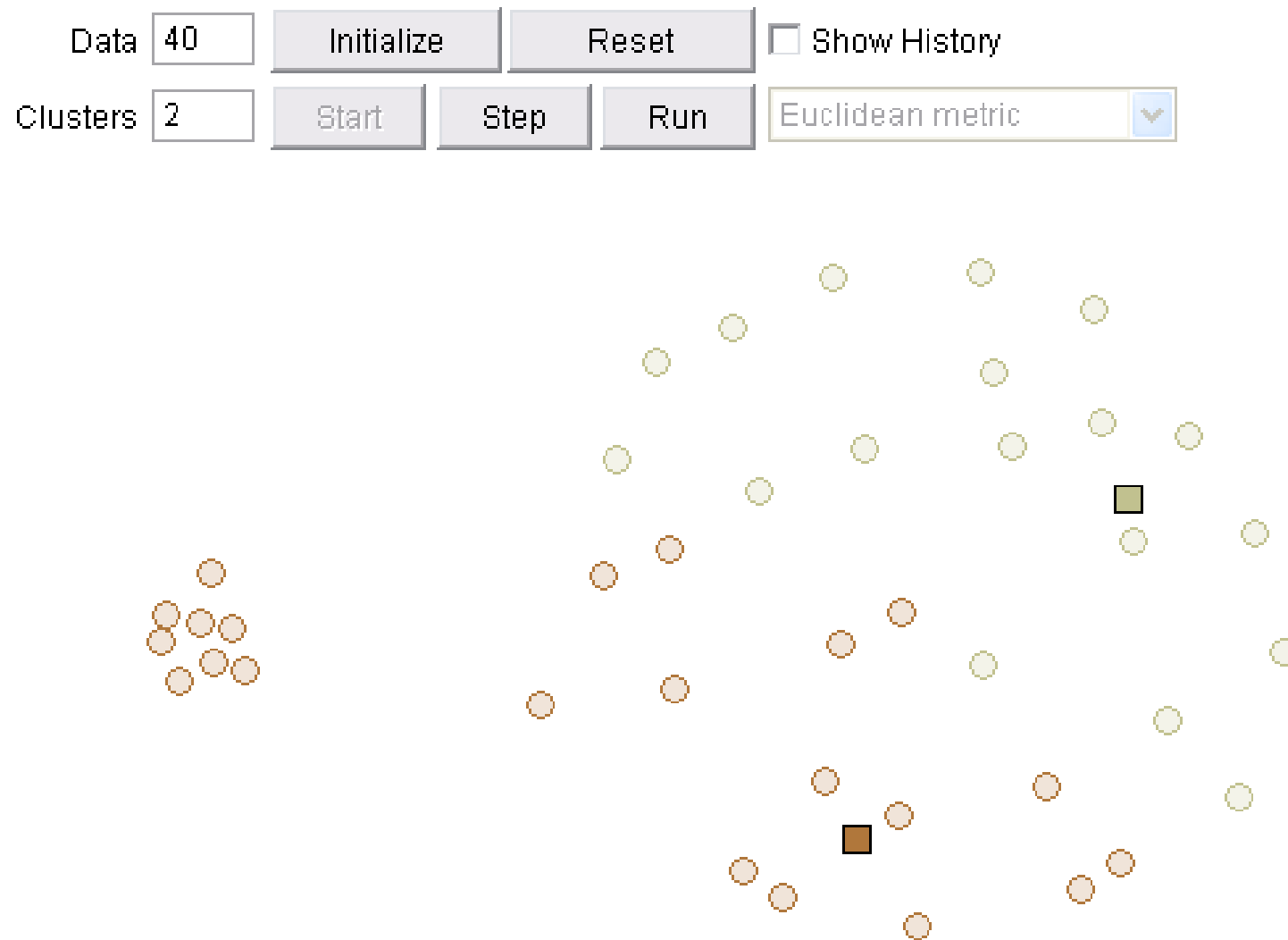


Ea zein den algoritmoaren jokabidea

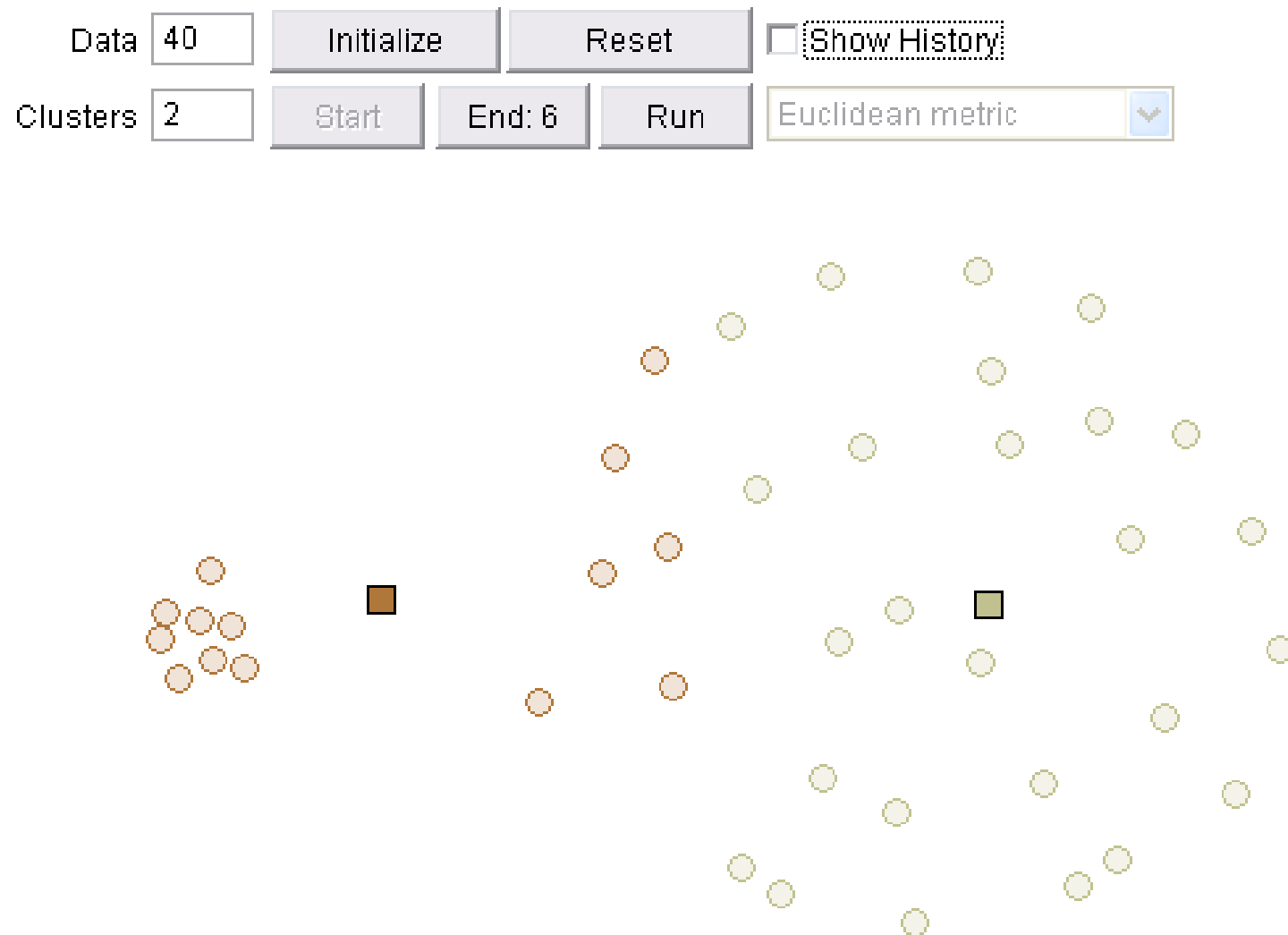
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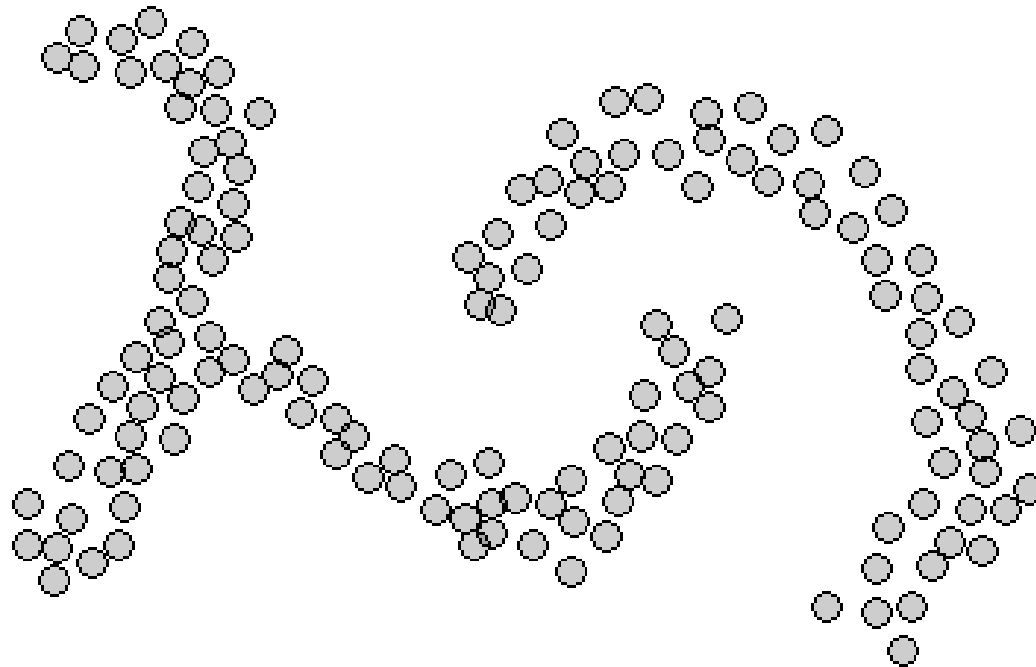


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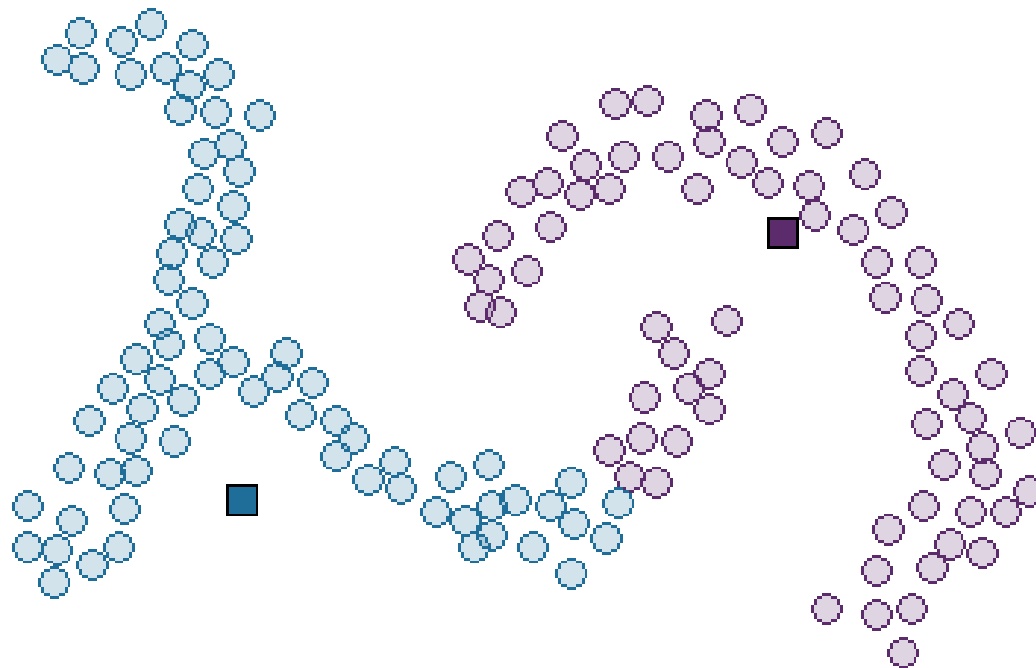
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Data	145	Initialize	Reset	☐ Show History		
Clusters	2	Start	Step	Run	Euclidean metric	v



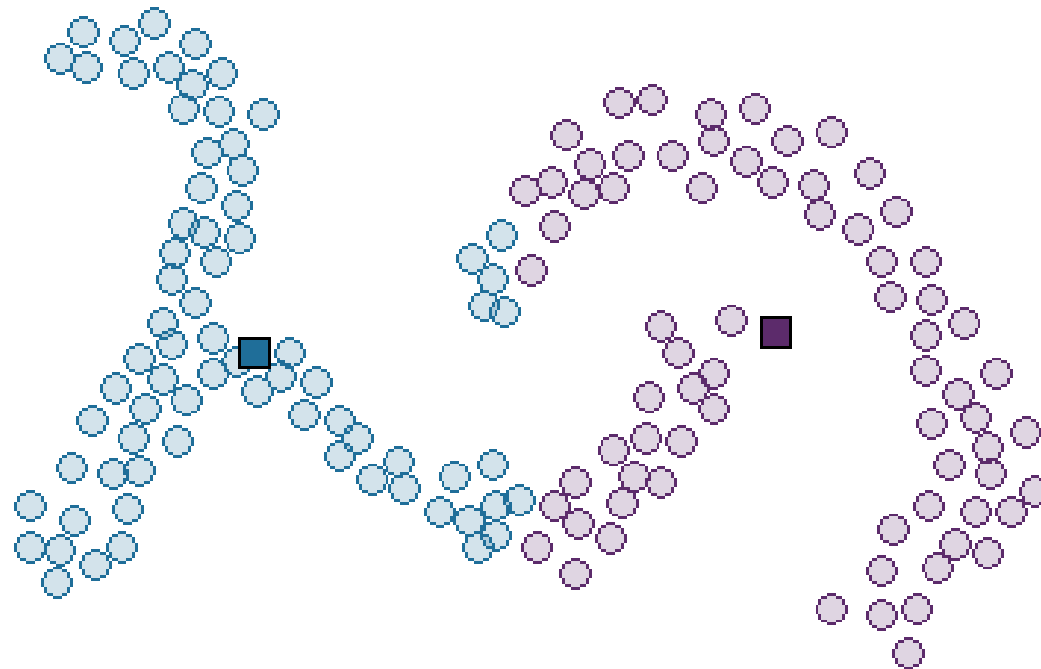
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Data	145	Initialize	Reset	☐ Show History
Clusters	2	Start	Step	Run
				Euclidean metric



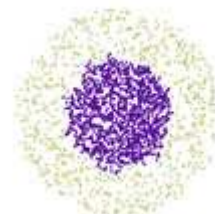
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Data	145	Initialize	Reset	☐ Show History		
Clusters	2	Start	End: 1	Run	Euclidean metric	v



Partiketarako algoritmoa: k-means

Irudi hauetan 2 sail edo cluster ikusten dugu.



Clusteringa egiteko ohiko tekniken atzean dagoena zera da, *clusterrak* esferikoak eta trinkotasun berdinekoak direla.

Clusterrak ez direnean halakoak algoritmoek ez dute emaitza egokirik ematen.

Giza-senarekin bat etortzeko bestelako teknikak behar dira.

Hierarkiarako algoritmoa

Algoritmoa (zatikorra):

- Sail bakarra duen partiketa (Ω) hartu abiapuntutzat.
- Aurreko partiketaren bi sail *ezantzekoenak* **zatitu**, hau da, partiketa berri bat osatu.
- Partiketa berria bakarkako sailez osatuta badago, bukatu, bestela itzuli 2. urratsera.

Konputazionalki garestia, ez bada bilaketa murrizten

Hierarkiarako algoritmoa

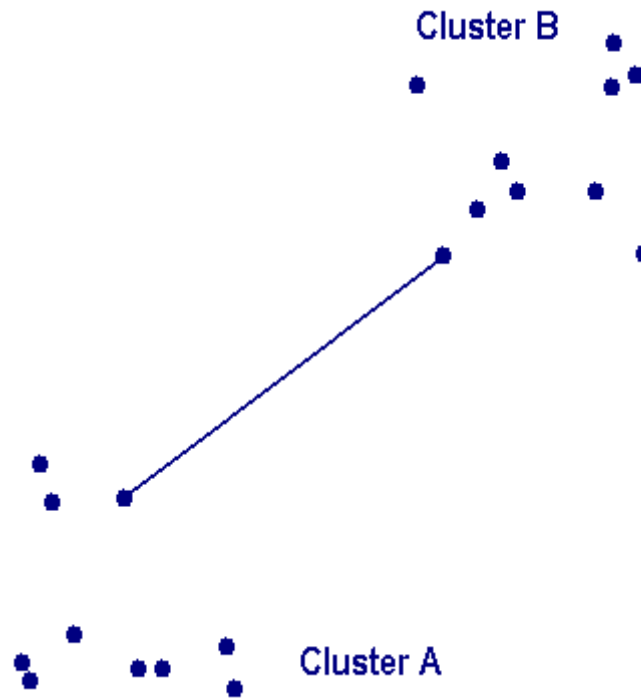
Algoritmoa (elkarkorra):

- Bakarkako partiketa edo sailkapena hartu abiapuntutzat (objektu bakoitzak sail bat osatzen du).
- Aurreko partiketaren bi sail gertuenak (*distantzia*) edo *antzekoenak elkartu*, hau da, partiketa berri bat osatu.
- Partiketa berria sail bakar batez osatuta badago (Ω), bukatu, bestela itzuli 2. urratsera.

Hierarkiarako algoritmoa

Sailen arteko distantzia:

single-linkage



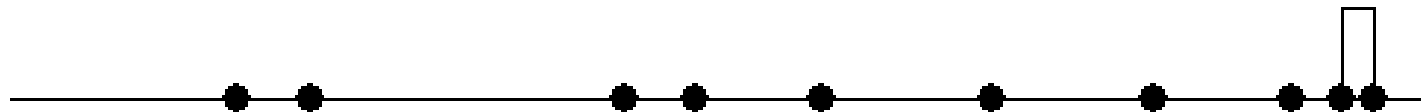
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Data	10	Initialize	Reset
Method	Single-linkage 	Step	Run



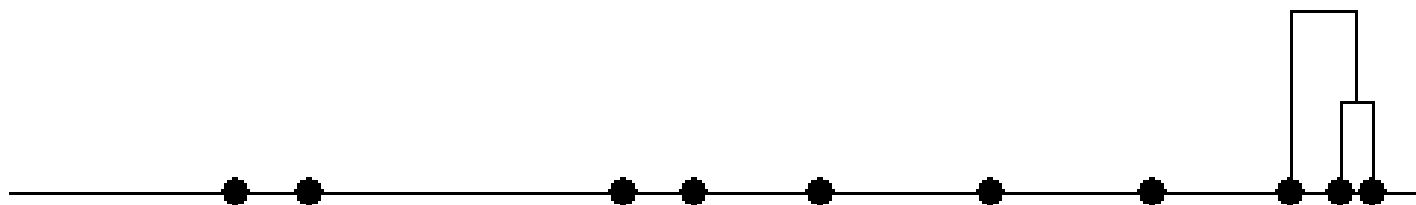
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Data	10	Initialize	Reset
Method	Single-linkage 	1	Run



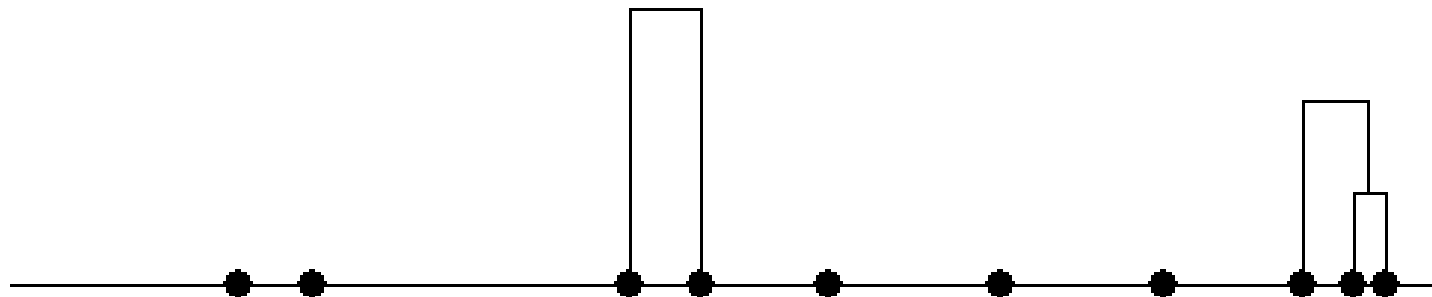
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Data	10	Initialize	Reset
Method	Single-linkage v	2	Run



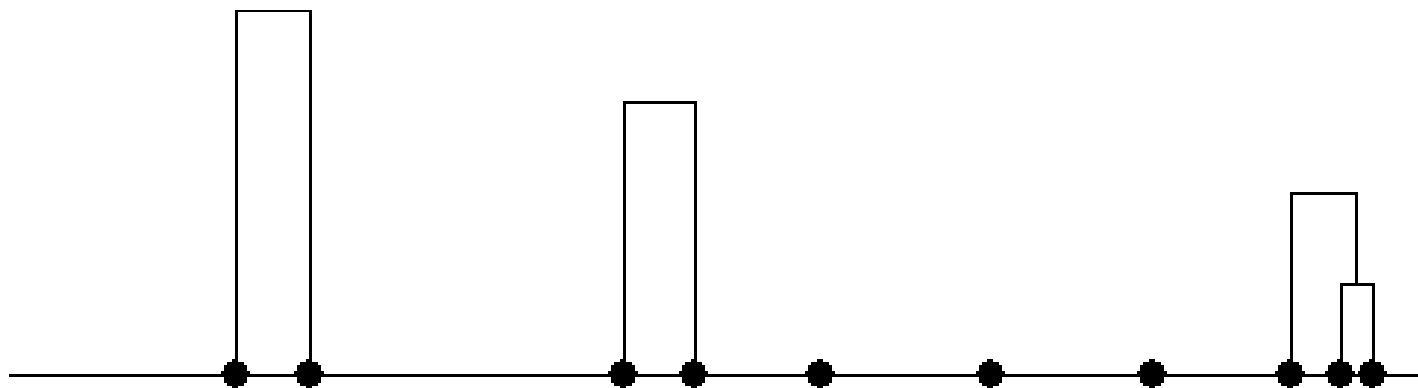
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Data	10	Initialize	Reset
Method	Single-linkage ▼	3	Run



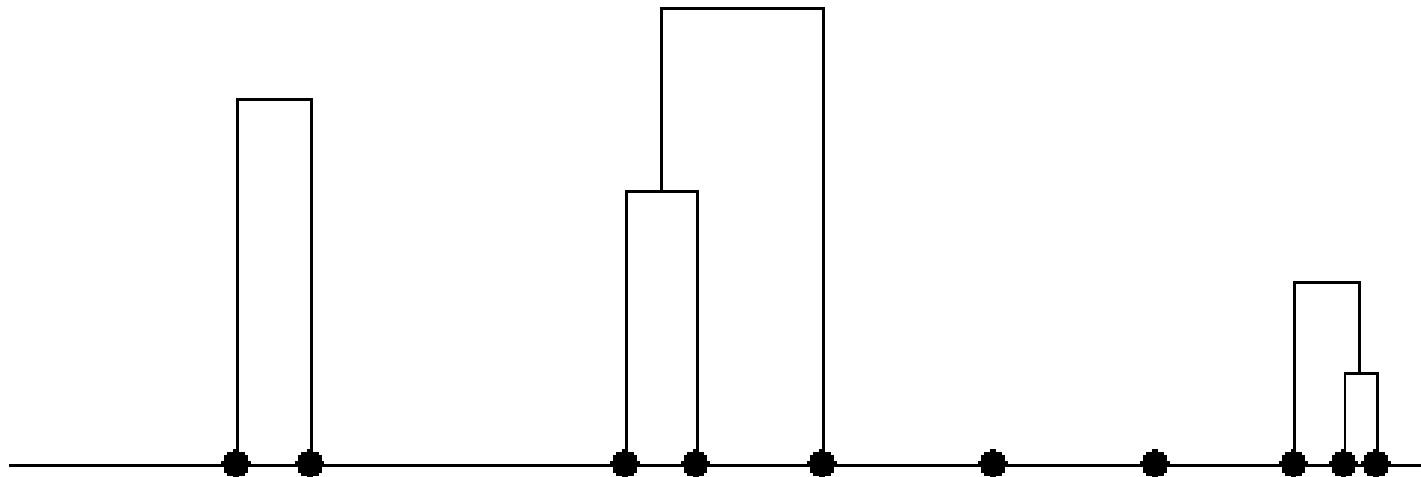
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Data	10	Initialize	Reset
Method	Single-linkage v	4	Run



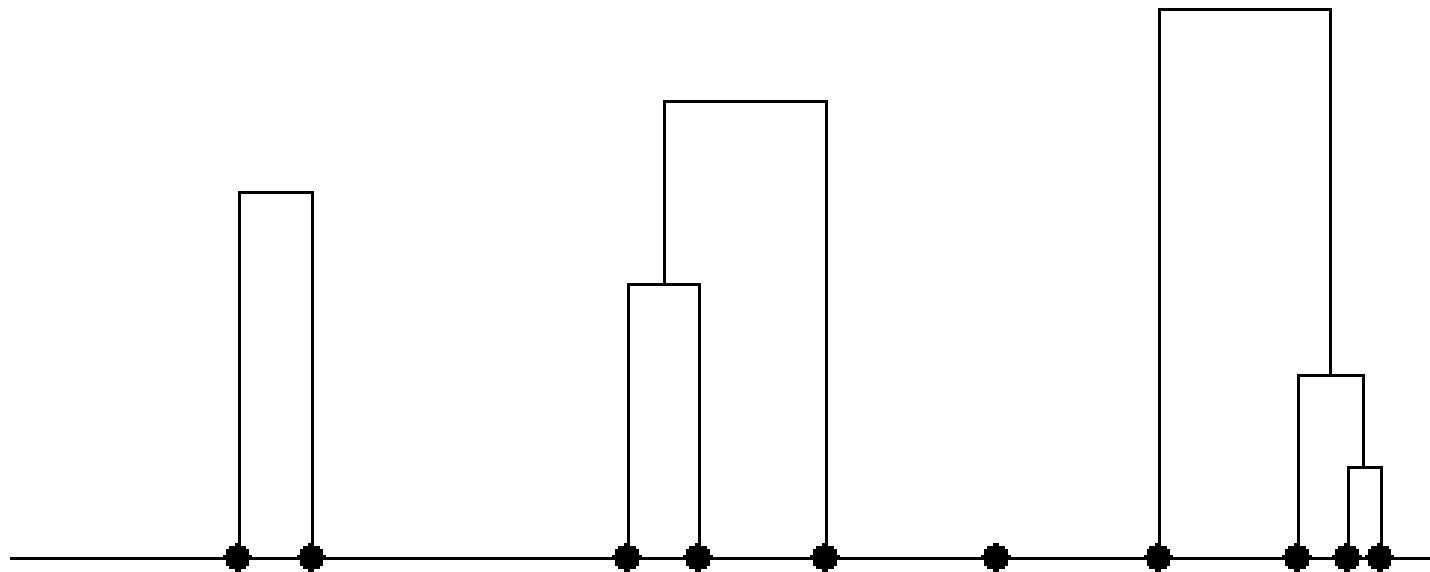
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Data	10	Initialize	Reset
Method	Single-linkage v	5	Run



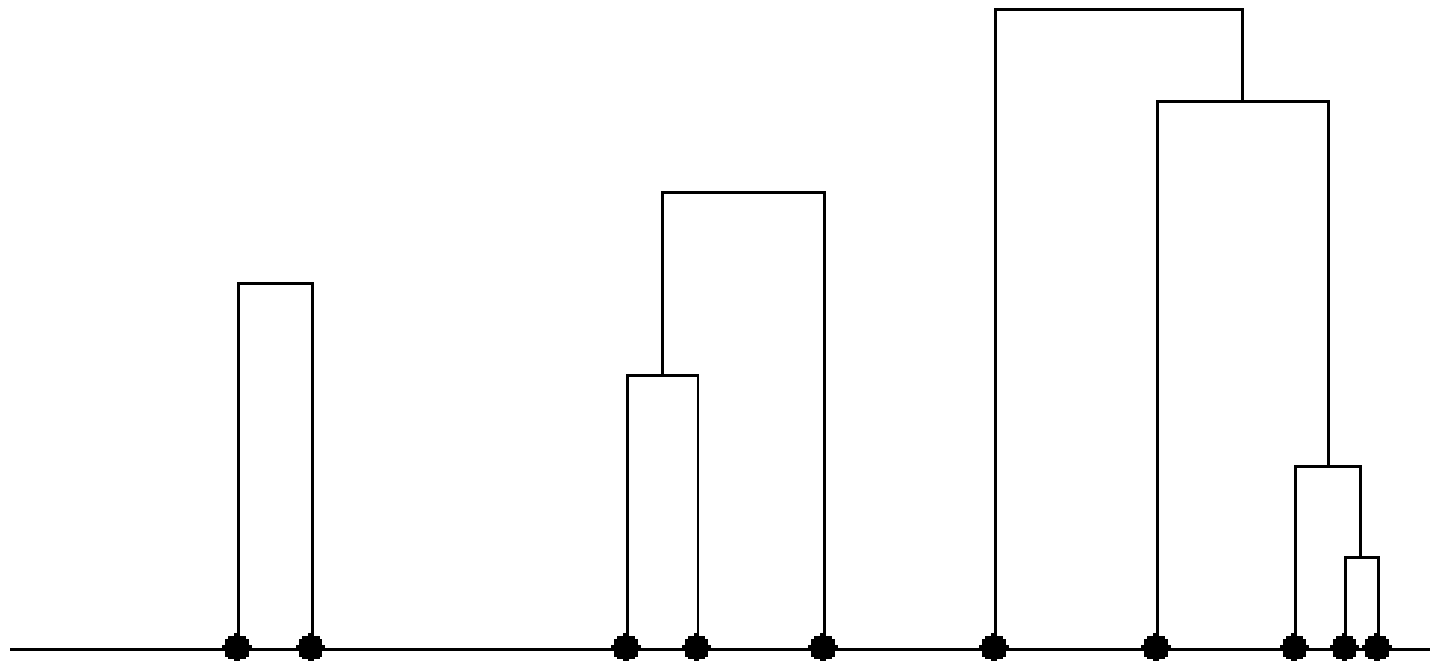
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Data	10	Initialize	Reset
Method	Single-linkage	6	Run



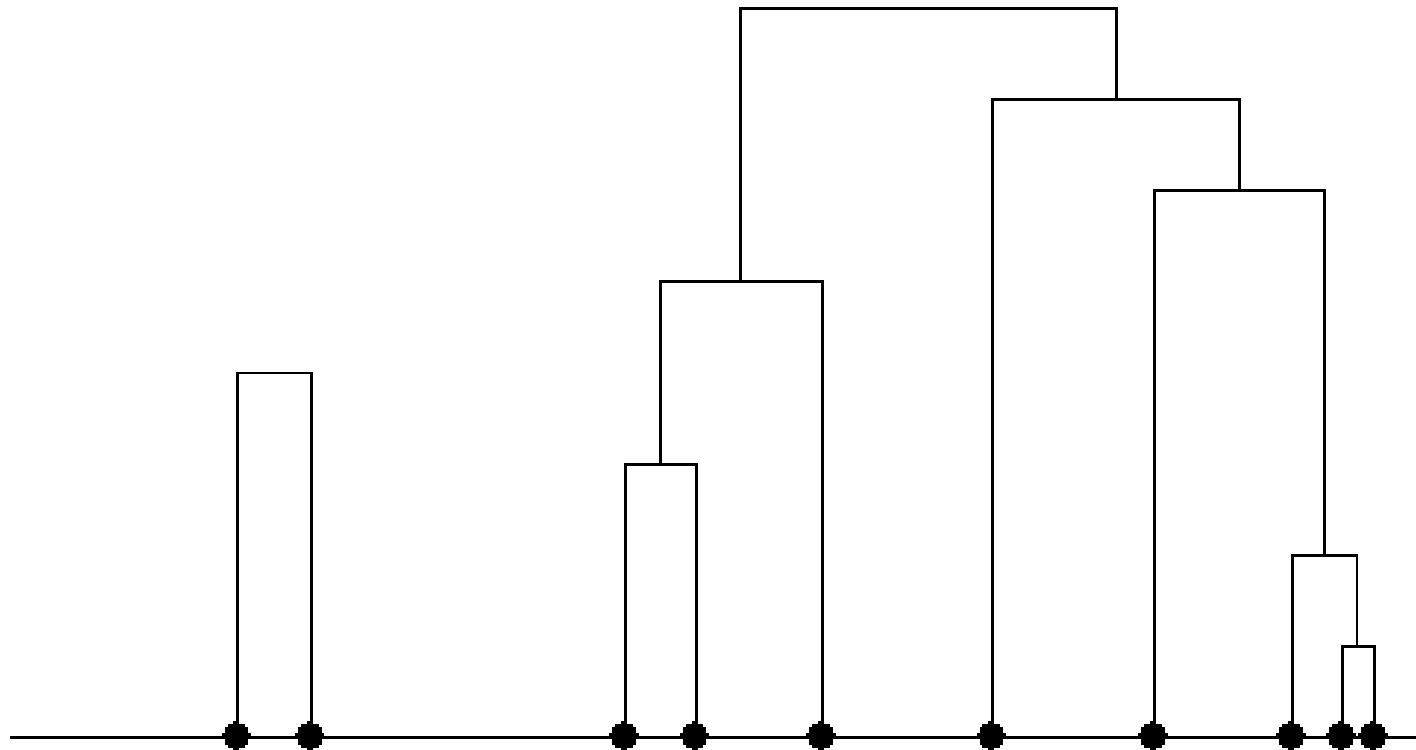
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Data	10	Initialize	Reset
Method	Single-linkage ▼	7	Run



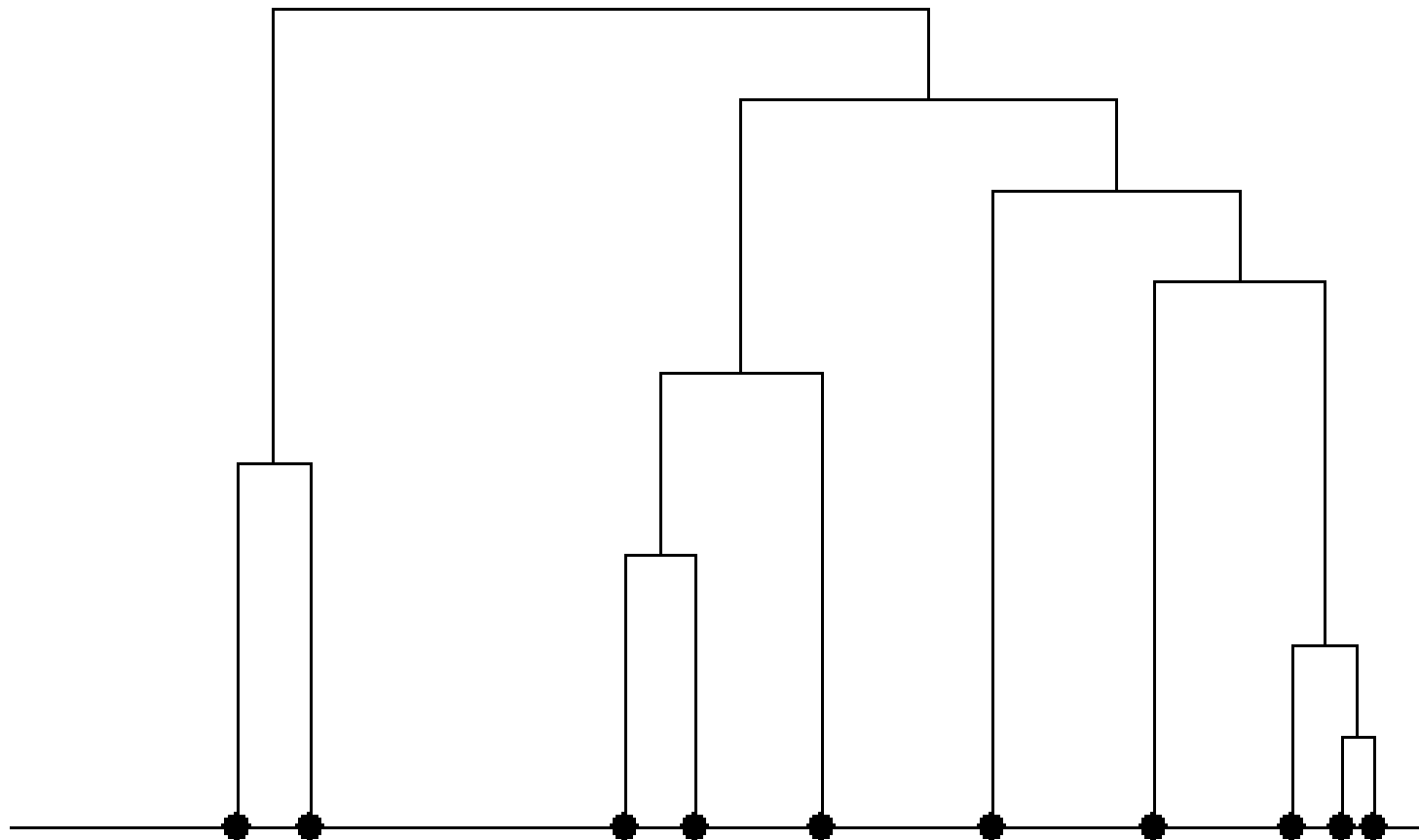
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Data	10	Initialize	Reset
Method	Single-linkage ▼	8	Run



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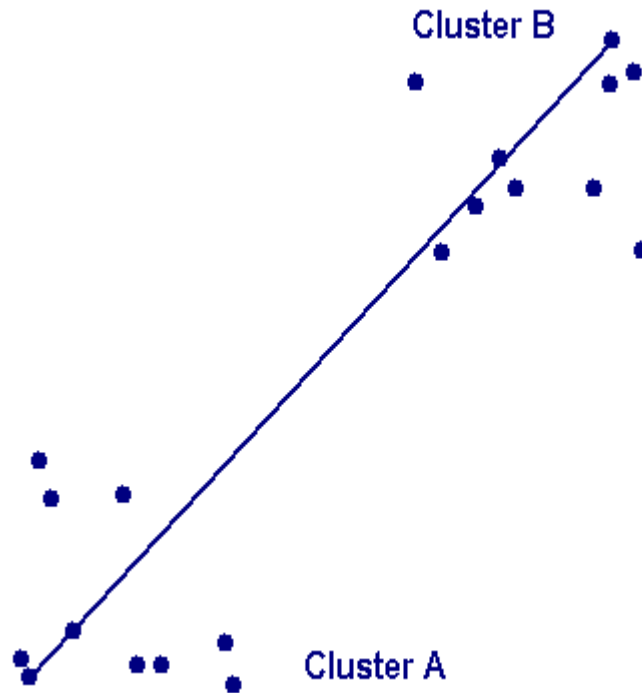
Data	10	Initialize	Reset
Method	Single-linkage v	End: 9	Run



Hierarkiarako algoritmoa

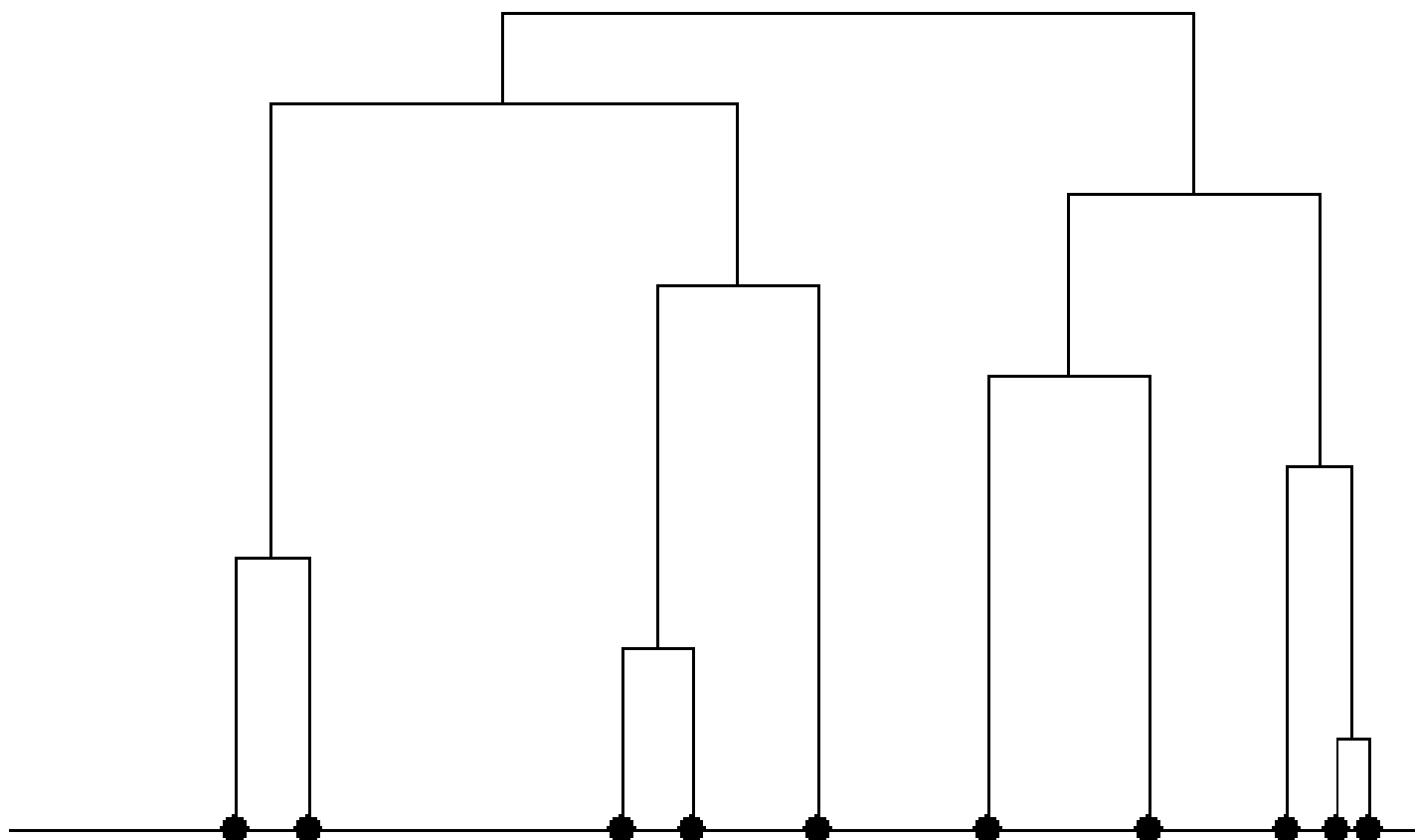
Sailen arteko distantzia:

complete-linkage



http://home.dei.polimi.it/matteucc/Clustering/tutorial_html/AppletH.html

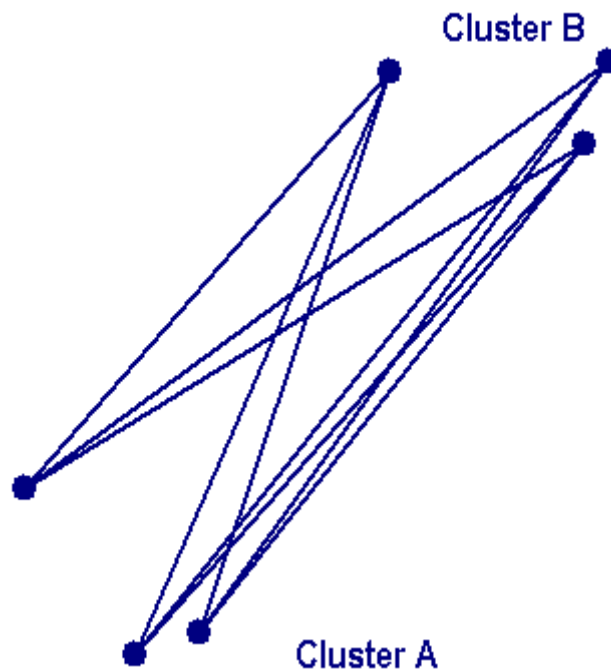
Data	10	Initialize	Reset
Method	Complete-linkage v	End: 9	Run



Hierarkiarako algoritmoa

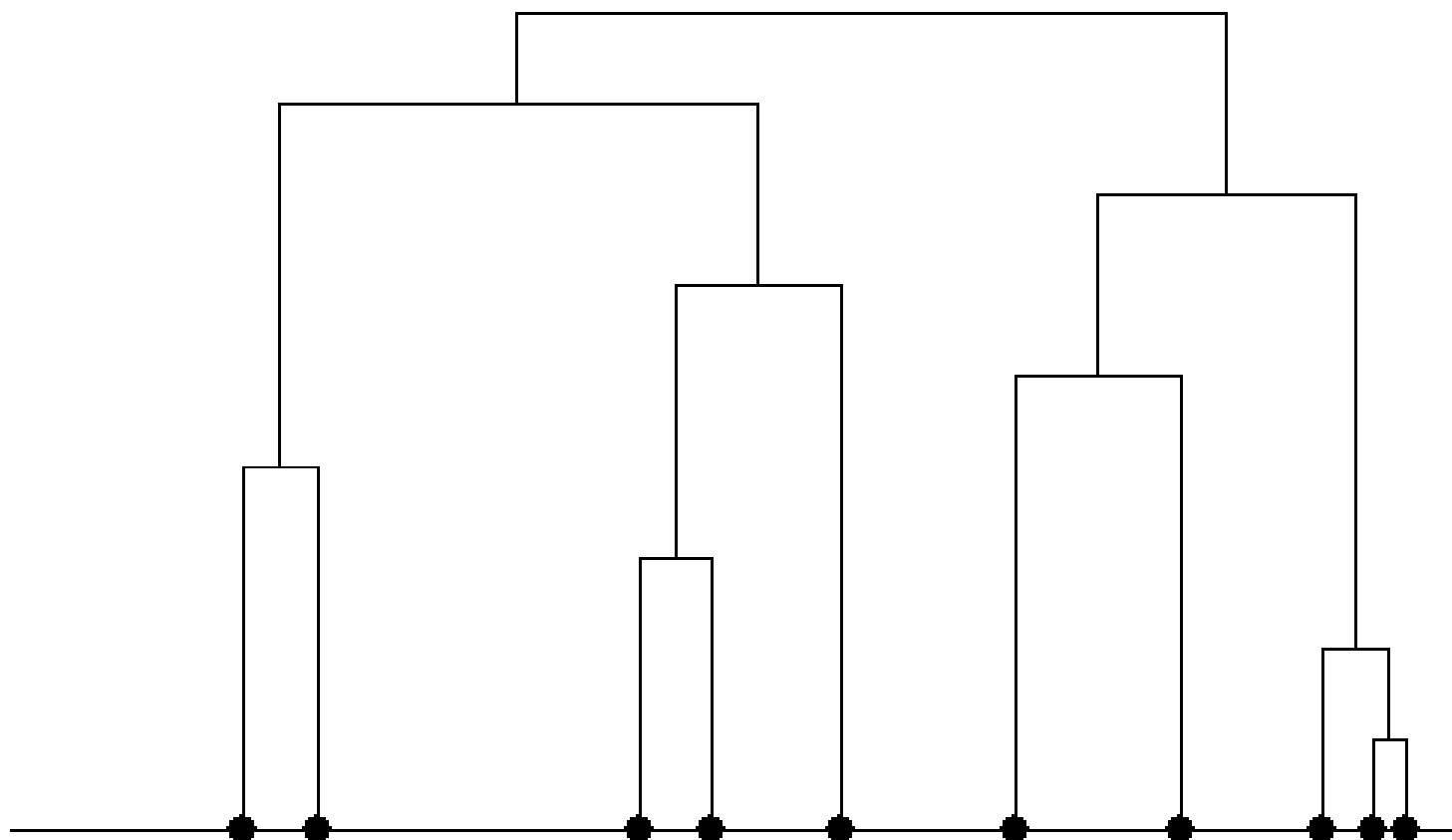
Sailen arteko distantzia:

average-linkage



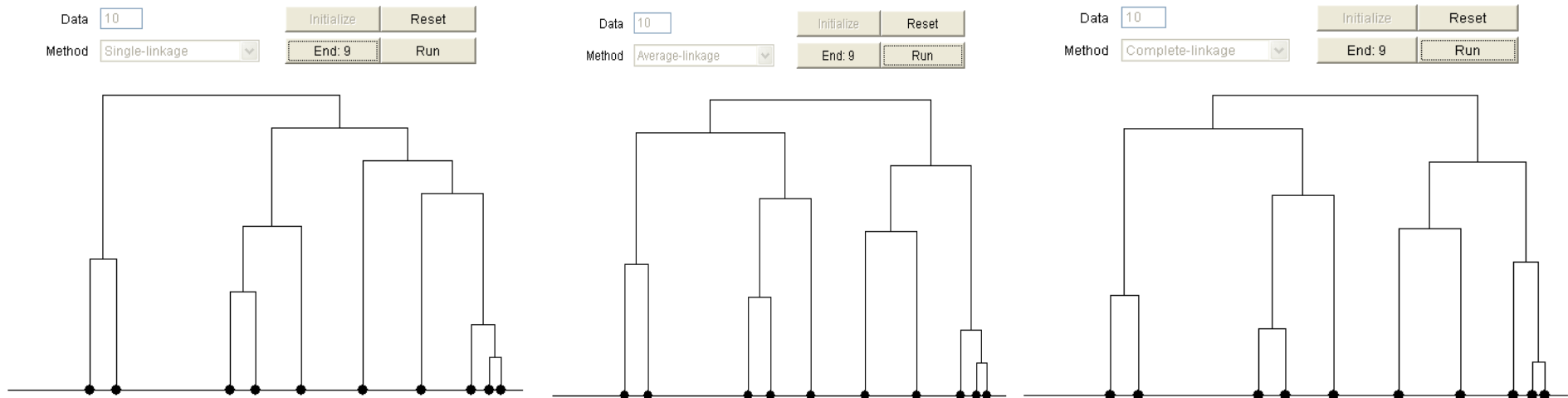
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Data	10	Initialize	Reset
Method	Average-linkage ▼	End: 9	Run



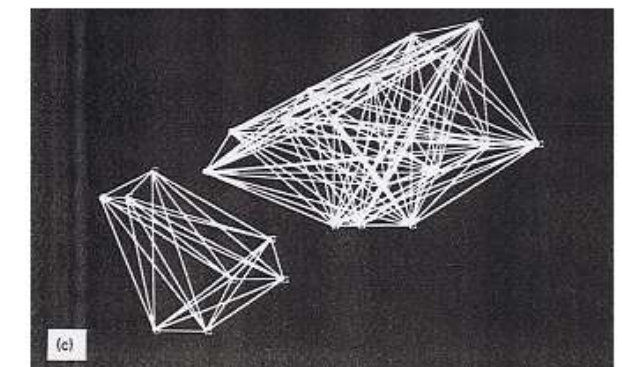
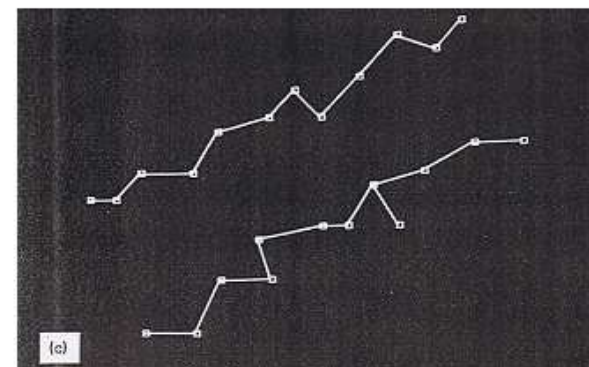
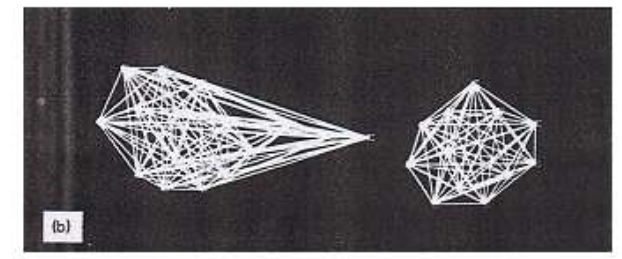
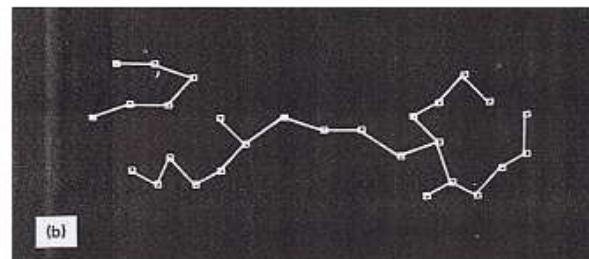
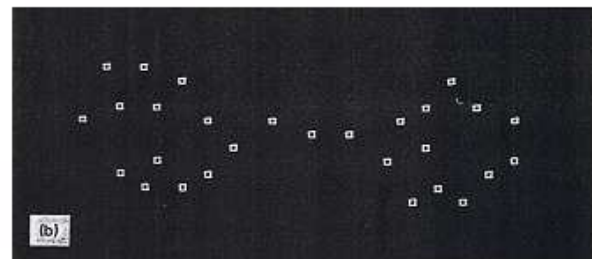
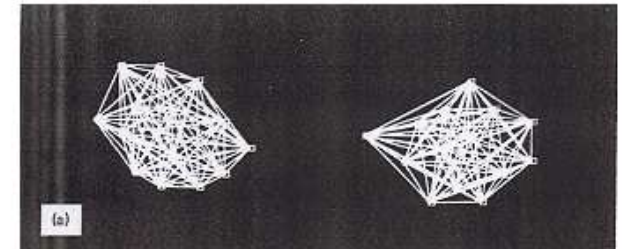
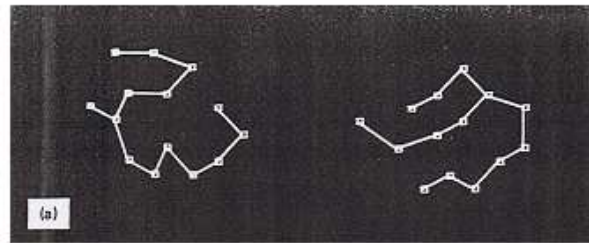
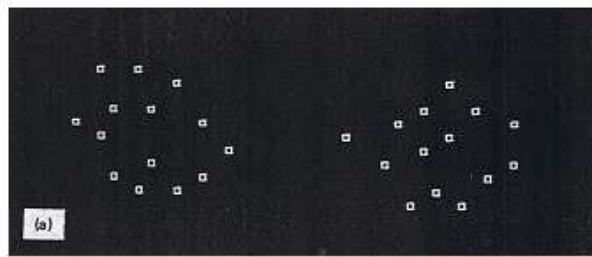
Hierarkiarako algoritmoa

Orokorrean, emaitzak ez dira berberak



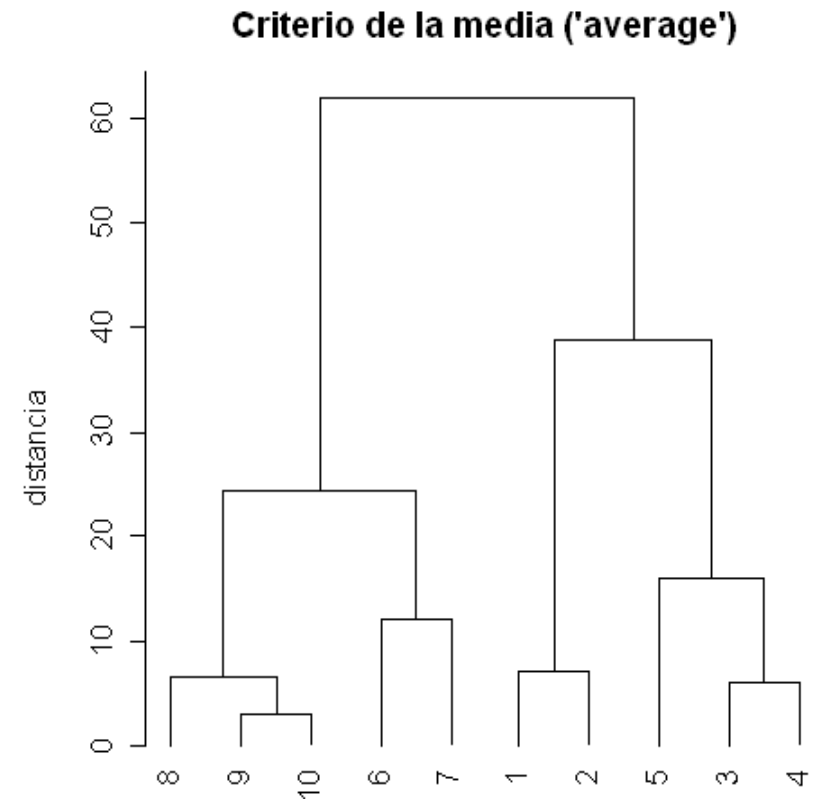
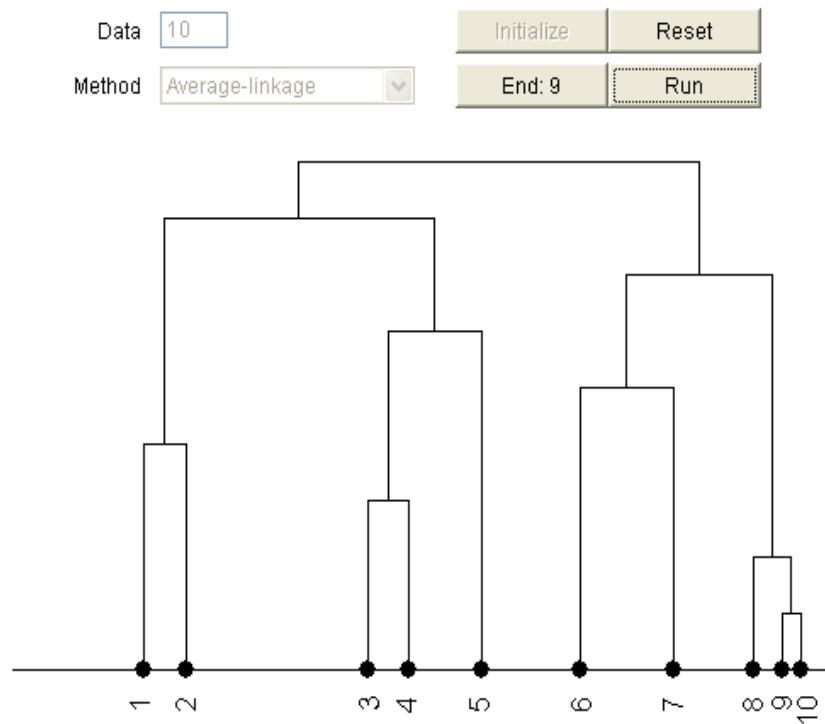
Hierarkiarako algoritmoa

Orokorrean, emaitzak ez dira berberak

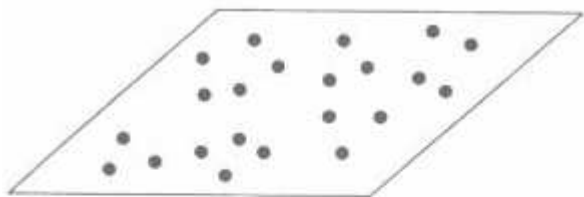


Hierarkiarako algoritmoa

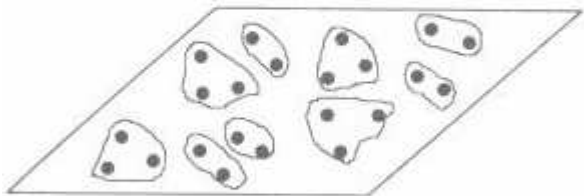
Indexadun hierarkia



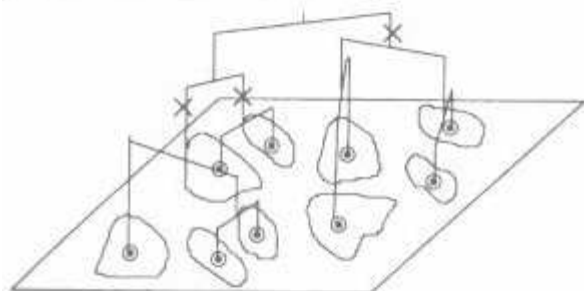
Datu-bolumen handia lantzeko sailakpen-estrategia



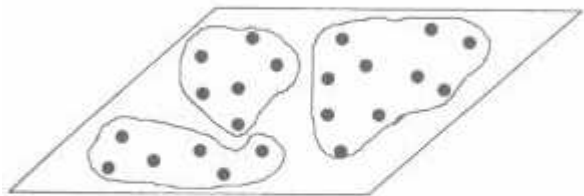
Datuak sailkapenaren aurretik



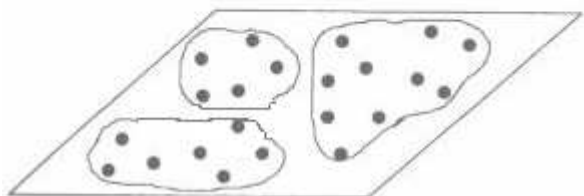
Hasierako partiketa azkarra: *k-means*



Partiketaren erdiguneen gaineko *hierarkia*



Dendrograma moztuz lortutako *partiketa*



Azkeneko partiketa: sendotu *berresleituz*

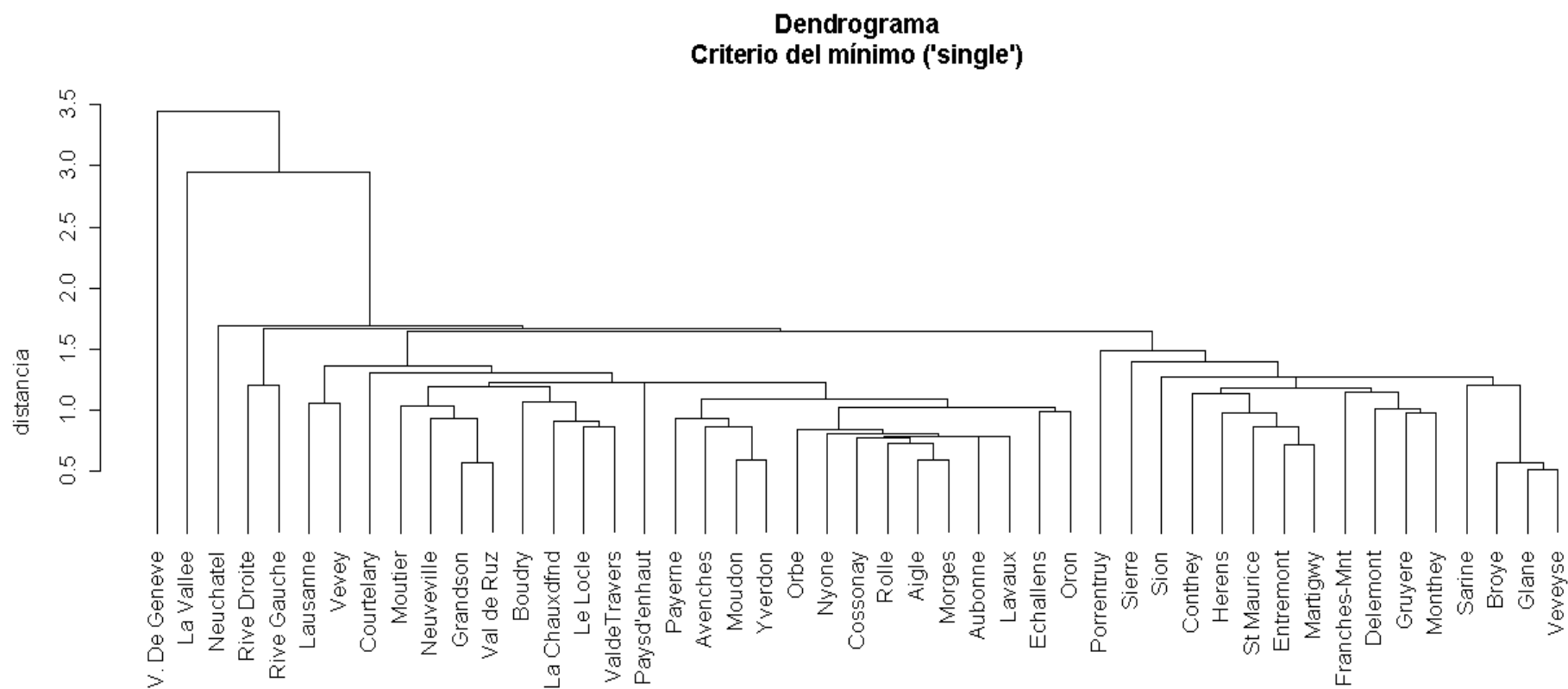
Suitza, 1888: 47 herrialde frankofono

Zenbait adierazle sozioekonomiko (%) eta emankortasuna

X1 Fertility X2 Agriculture X3 Examination
X4 Education X5 Catholic X6 Infant.Mortality

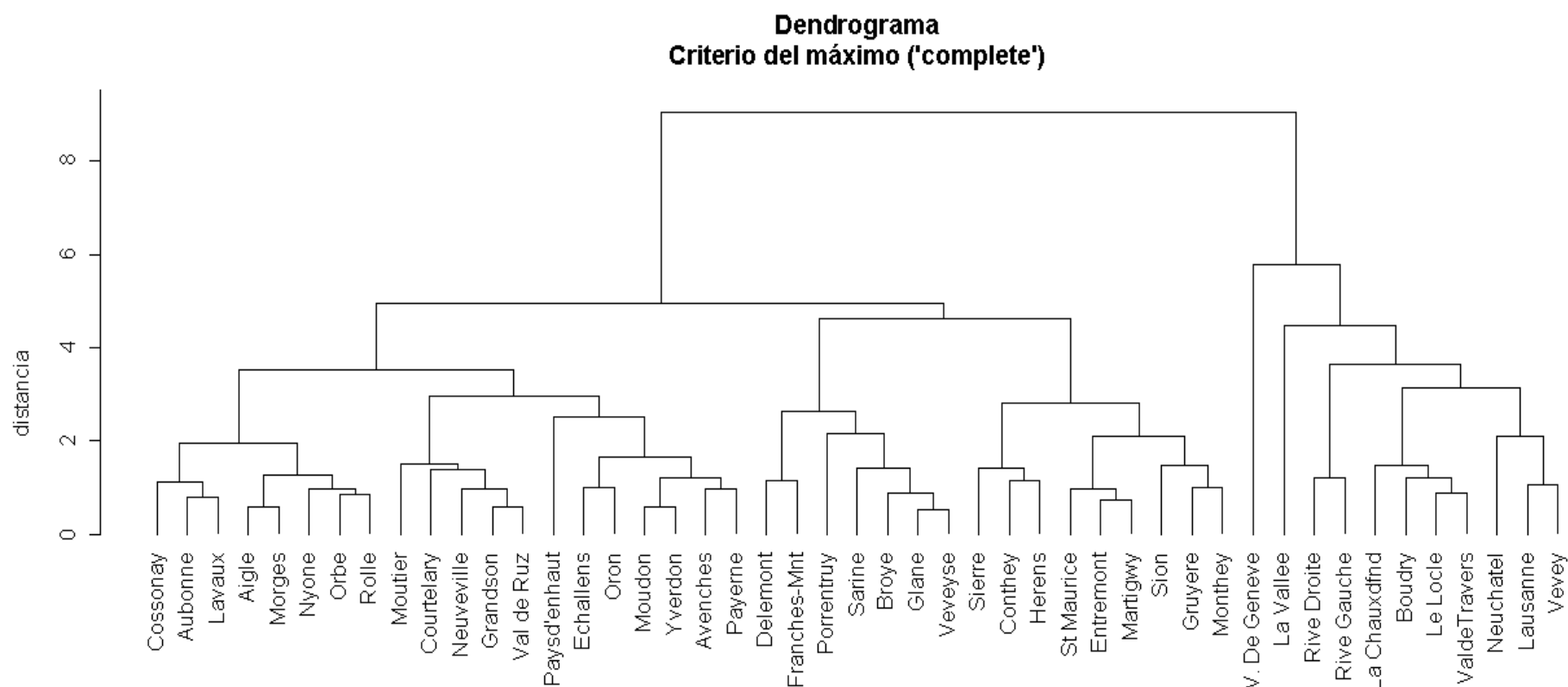
	X1	X2	X3	X4	X5	X6		X1	X2	X3	X4	X5	X6
Courtelary	80.2	17.0	15	12	9.96	22.2	Oron	72.5	71.2	12	1	2.40	21.0
Delemont	83.1	45.1	6	9	84.84	22.2	Payerne	74.2	58.1	14	8	5.23	23.8
Franches-Mnt	92.5	39.7	5	5	93.40	20.2	Paysd'enhaut	72.0	63.5	6	3	2.56	18.0
Moutier	85.8	36.5	12	7	33.77	20.3	Rolle	60.5	60.8	16	10	7.72	16.3
Neuveville	76.9	43.5	17	15	5.16	20.6	Vevey	58.3	26.8	25	19	18.46	20.9
Porrentruy	76.1	35.3	9	7	90.57	26.6	Yverdon	65.4	49.5	15	8	6.10	22.5
Broye	83.8	70.2	16	7	92.85	23.6	Conthey	75.5	85.9	3	2	99.71	15.1
Glane	92.4	67.8	14	8	97.16	24.9	Entremont	69.3	84.9	7	6	99.68	19.8
Gruyere	82.4	53.3	12	7	97.67	21.0	Herens	77.3	89.7	5	2	100.00	18.3
Sarine	82.9	45.2	16	13	91.38	24.4	Martigwy	70.5	78.2	12	6	98.96	19.4
Veveyse	87.1	64.5	14	6	98.61	24.5	Monthey	79.4	64.9	7	3	98.22	20.2
Aigle	64.1	62.0	21	12	8.52	16.5	St Maurice	65.0	75.9	9	9	99.06	17.8
Aubonne	66.9	67.5	14	7	2.27	19.1	Sierre	92.2	84.6	3	3	99.46	16.3
Avenches	68.9	60.7	19	12	4.43	22.7	Sion	79.3	63.1	13	13	96.83	18.1
Cossonay	61.7	69.3	22	5	2.82	18.7	Boudry	70.4	38.4	26	12	5.62	20.3
Echallens	68.3	72.6	18	2	24.20	21.2	La Chauxdfnd	65.7	7.7	29	11	13.79	20.5
Grandson	71.7	34.0	17	8	3.30	20.0	Le Locle	72.7	16.7	22	13	11.22	18.9
Lausanne	55.7	19.4	26	28	12.11	20.2	Neuchatel	64.4	17.6	35	32	16.92	23.0
La Vallee	54.3	15.2	31	20	2.15	10.8	Val de Ruz	77.6	37.6	15	7	4.97	20.0
Lavaux	65.1	73.0	19	9	2.84	20.0	ValdeTravers	67.6	18.7	25	7	8.65	19.5
Morges	65.5	59.8	22	10	5.23	18.0	V. De Geneve	35.0	1.2	37	53	42.34	18.0
Moudon	65.0	55.1	14	3	4.52	22.4	Rive Droite	44.7	46.6	16	29	50.43	18.2
Nyone	56.6	50.9	22	12	15.14	16.7	Rive Gauche	42.8	27.7	22	29	58.33	19.3
Orbe	57.4	54.1	20	6	4.20	15.3							

```
> dswiss <- dist(swiss)
> hsingle <- hclust(dswiss, method="single")
> plot(hsingle, hang=-1, xlab="", ylab="distancia",
      main="Dendrograma\nCriterio del mínimo ('single')")
```



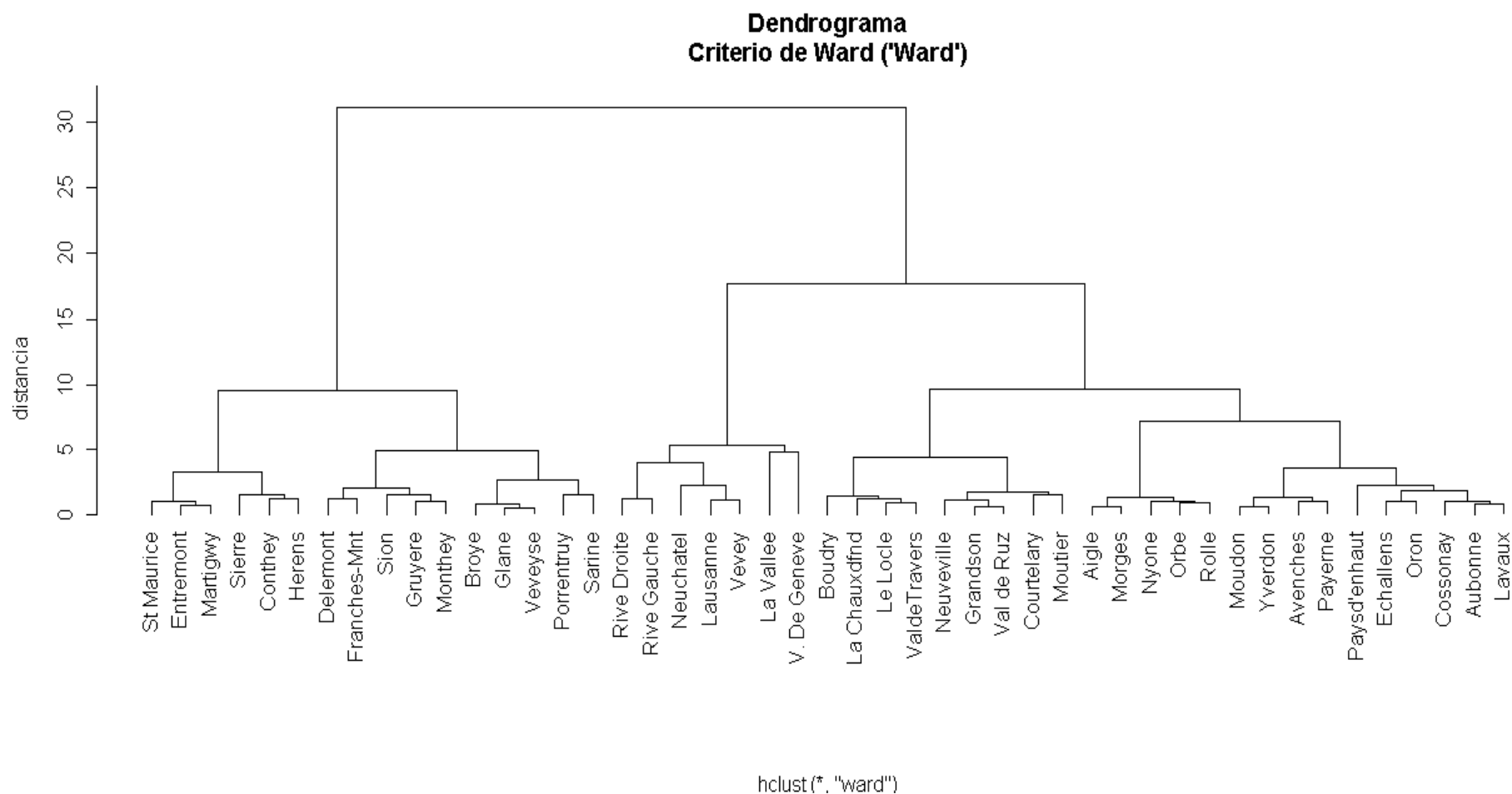
hclust(*, "single")

```
> dswiss <- dist(swiss)
> hcomplete <- hclust(dswiss, method="complete")
> plot(hcomplete, hang=-1, xlab="", ylab="distancia",
      main="Dendrograma\nCriterio del máximo ('complete')")
```

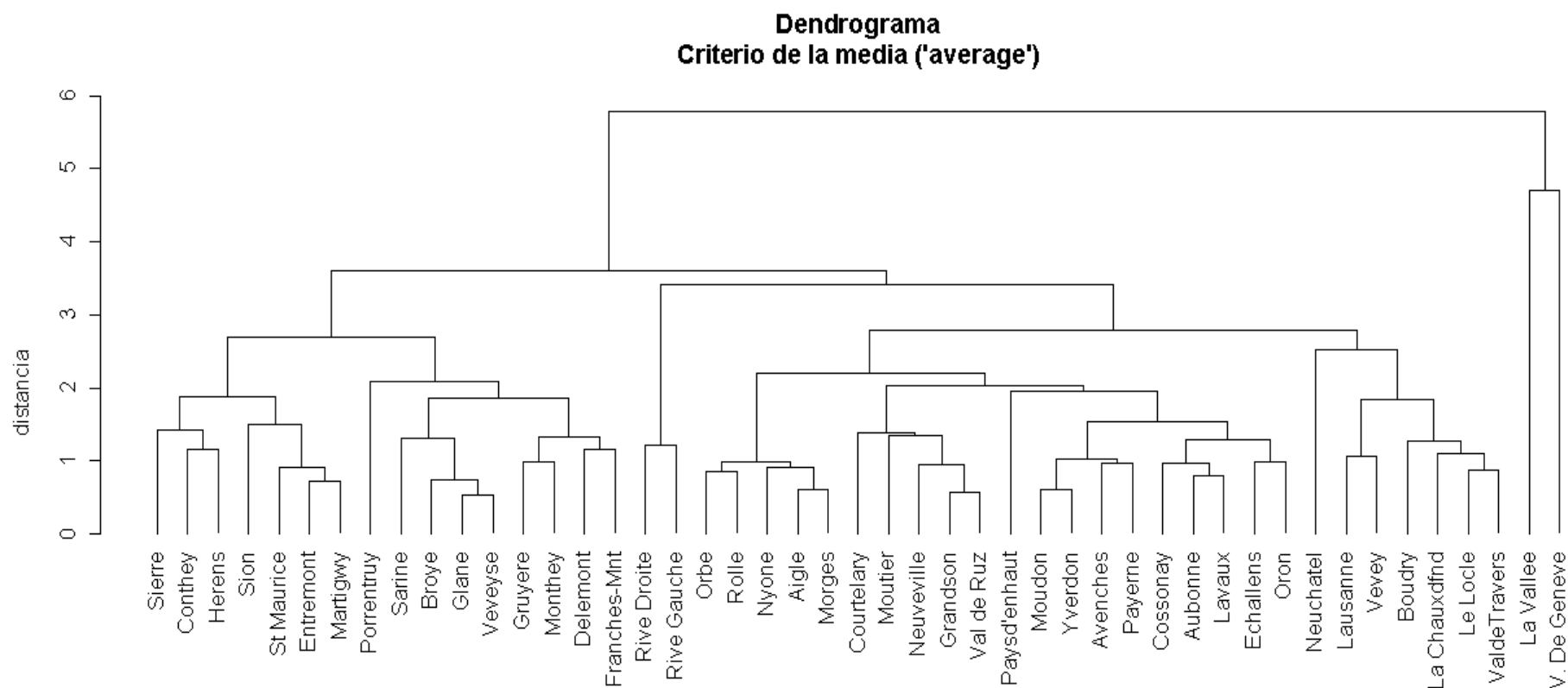


hclust(*, "complete")

```
> dswiss <- dist(swiss)
> hward <- hclust(dswiss, method="ward")
> plot(hward, hang=-1, xlab="", ylab="distancia",
      main="Dendrograma\nCriterio de Ward ('ward')")
```

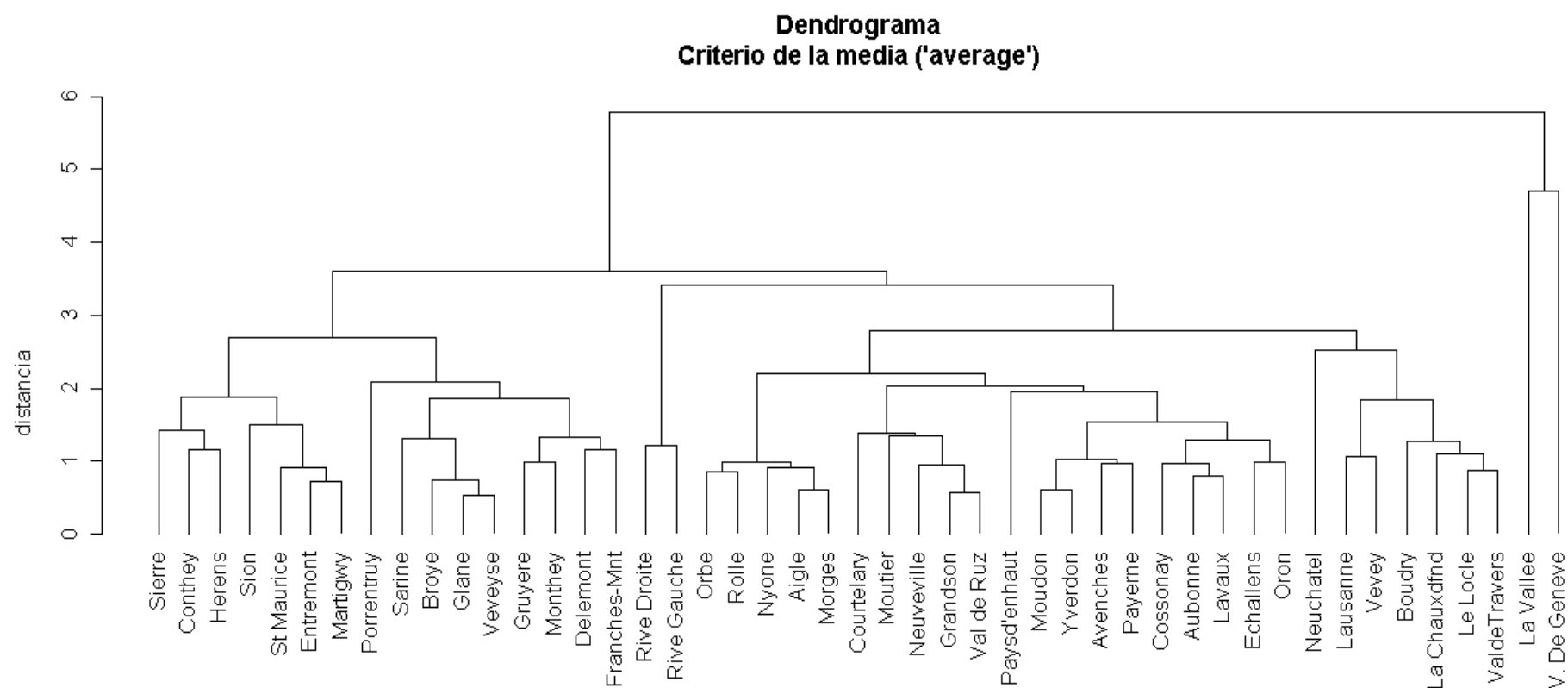


```
> dswiss <- dist(swiss)
> haverage <- hclust(dswiss, method="average")
> plot(haverage, hang=-1, xlab="", ylab="distancia",
      main="Dendrograma\nCriterio de la media ('average')")
```



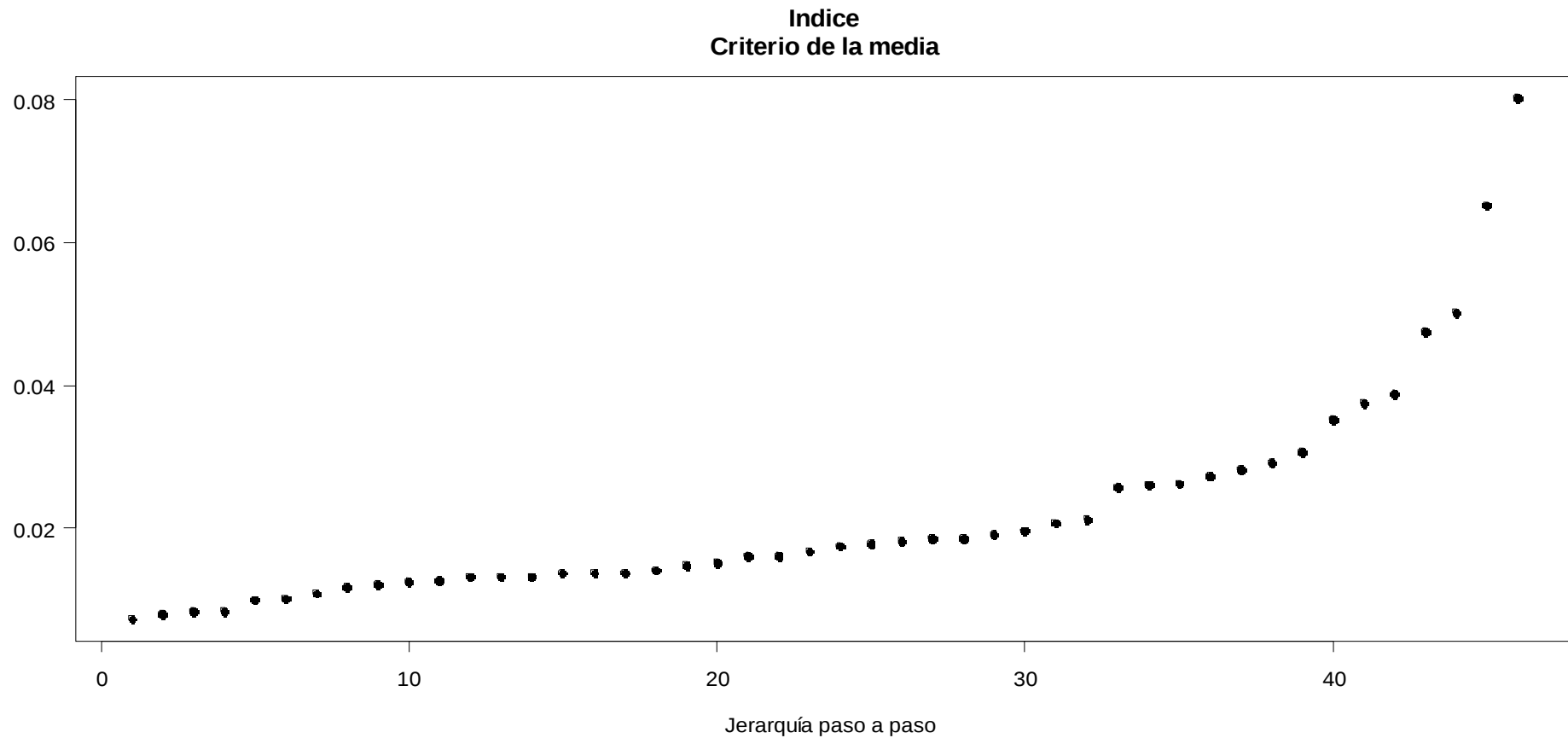
hclust(*, "average")

```
> dswiss <- dist(swiss)
> haverage <- hclust(dswiss, method="average")
> plot(haverage, hang=-1, xlab="", ylab="distancia",
      main="Dendrograma\nCriterio de la media ('average')")
```

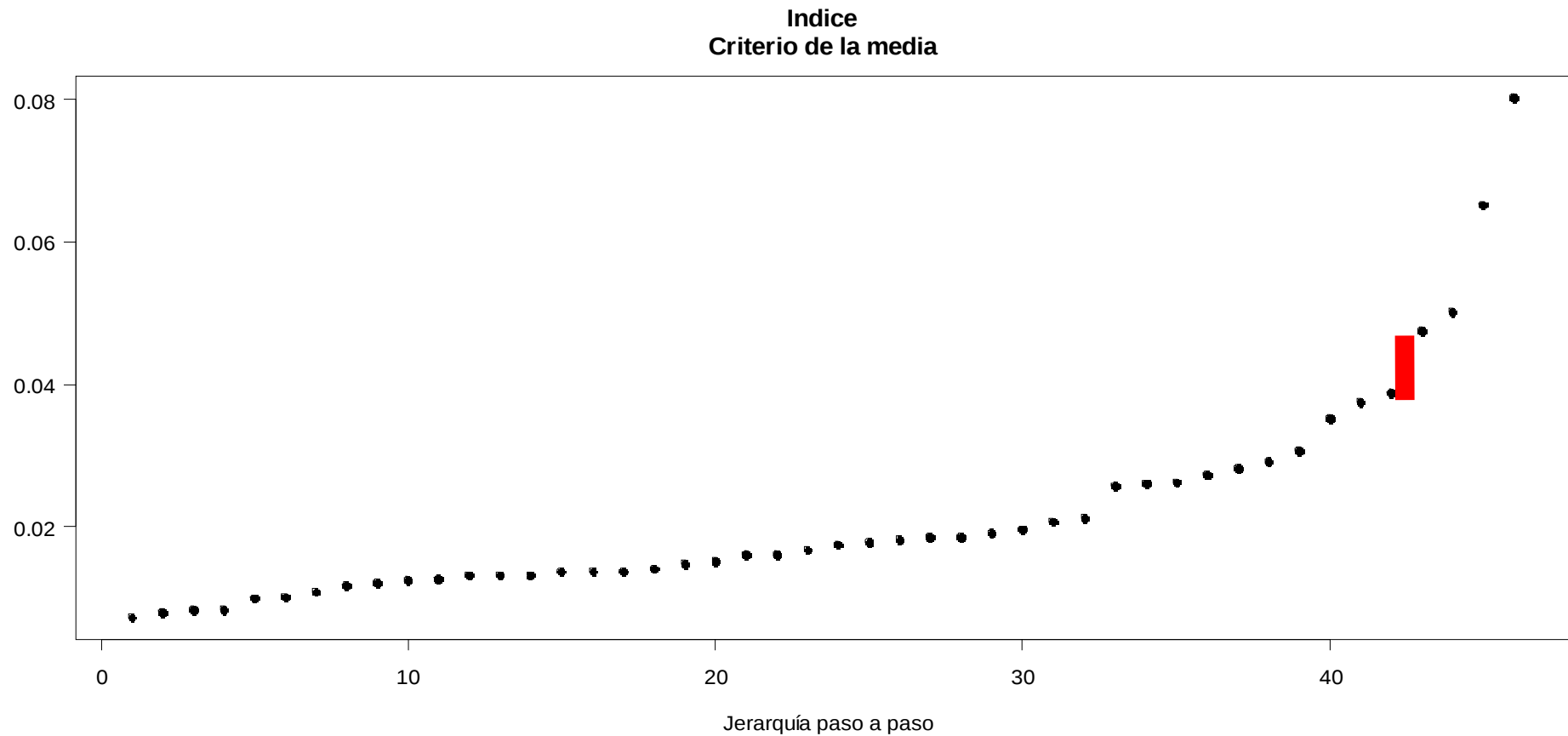


hclust(*, "average")

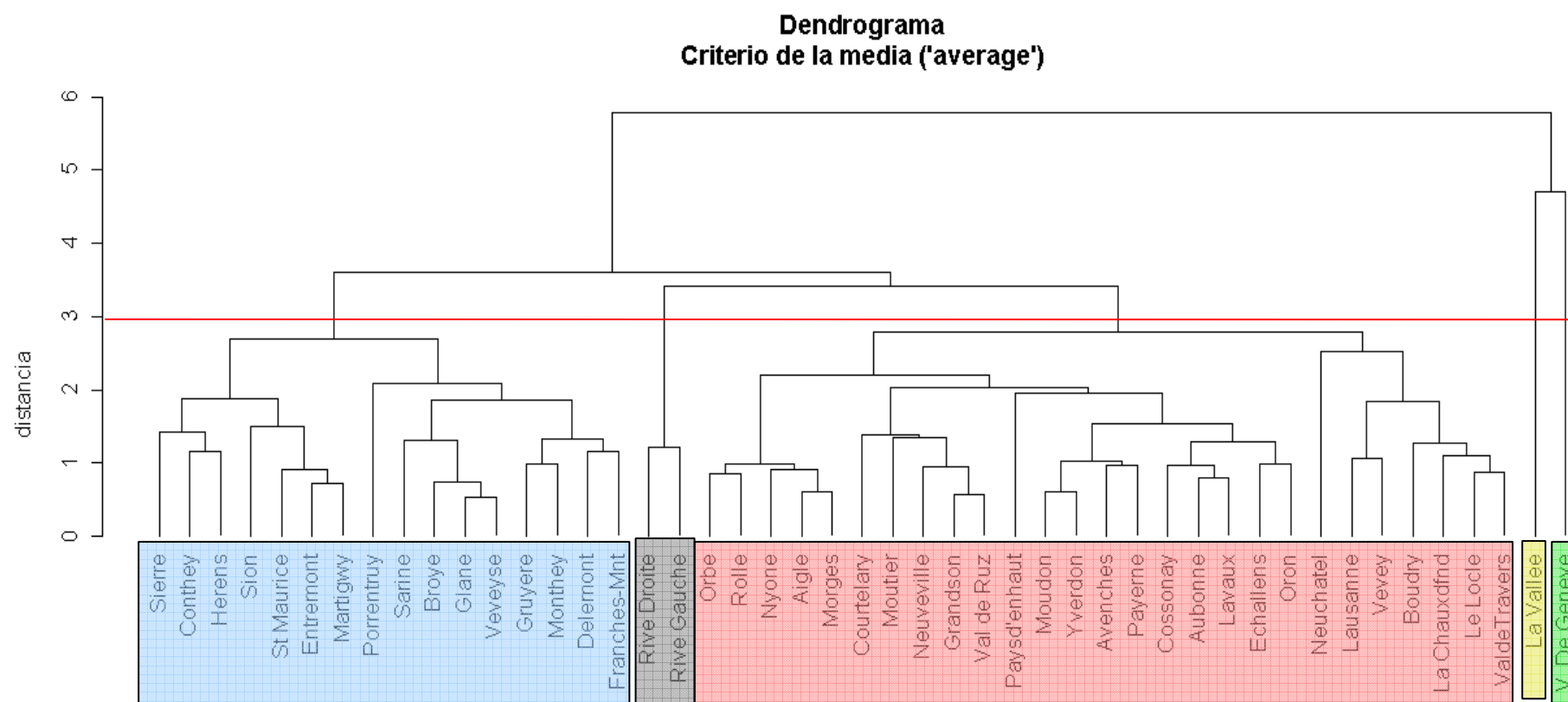
```
> plot(haverage$height/sum(haverage$height), las=1, pch=19,  
+       xlab="Jerarquía paso a paso", ylab="",  
+       main="Indice\nCriterio de la media")
```



```
> plot(haverage$height/sum(haverage$height), las=1, pch=19,  
+       xlab="Jerarquía paso a paso", ylab="",  
+       main="Indice\nCriterio de la media")
```



```
> dswiss <- dist(swiss)
> haverage <- hclust(dswiss, method="average")
> plot(haverage, hang=-1, xlab="", ylab="distancia",
      main="Dendrograma\nCriterio de la media ('average')")
```



hclust(*, "average")

```

> cuthaverage <- cutree(haverage, k=5)
> cuthaverage
> row.names(cutav) <- names(cuthaverage)
> cutav

```

Courtelary	1	Payerne	1
Delemont	2	Paysd'enhaut	1
Franches-Mnt	2	Rolle	1
Moutier	1	Vevey	1
Neuveville	1	Yverdon	1
Porrentruy	2	Conthey	2
Broye	2	Entremont	2
Glane	2	Herens	2
Gruyere	2	Martigwy	2
Sarine	2	Monthey	2
Veveyse	2	St Maurice	2
Aigle	1	Sierre	2
Aubonne	1	Sion	2
Avenches	1	Boudry	1
Cossonay	1	La Chauxdfnd	1
Echallens	1	Le Locle	1
Grandson	1	Neuchatel	1
Lausanne	1	Val de Ruz	1
La Vallee	3	ValdeTravers	1
Lavaux	1	V. De Geneve	4
Morges	1	Rive Droite	5
Moudon	1	Rive Gauche	5
Nyone	1		
Orbe	1		
Oron	1		

Balidazioa

```
> pca <- princomp(swiss, cor = TRUE, scores = TRUE)
```

Varianzas de las componentes construidas

```
> pca$sdev**2
      Comp.1      Comp.2      Comp.3
3.1997570 1.1883082 0.8476098
      Comp.4      Comp.5      Comp.6
0.4389287 0.2045337 0.1208626
> sum(pca$sdev**2)
[1] 6
```

Porcentajes de las *varianzas* de las componentes

```
> pca$sdev**2/sum(pca$sdev**2)
      Comp.1      Comp.2      Comp.3
0.53329283 0.19805137 0.14126830
      Comp.4      Comp.5      Comp.6
0.07315478 0.03408895 0.02014376
> sum((pca$sdev**2/sum(pca$sdev**2))[1:3])
[1] 0.8726125
```

3 componentes acumulan el **87.26%** de la *varianza total*

```

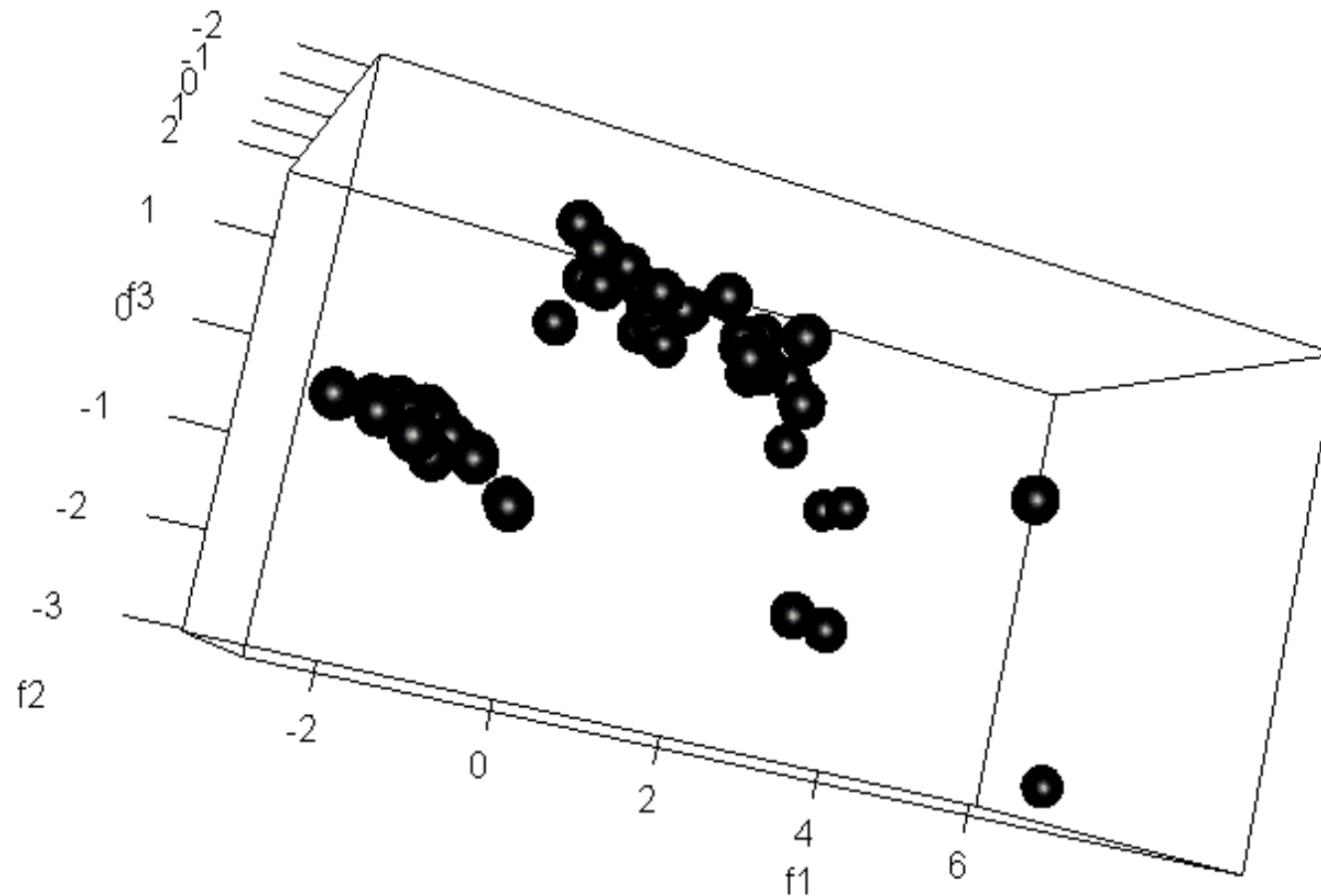
> pca <- princomp(swiss, cor = TRUE, scores = TRUE)
> attributes(pca)
$names
[1] "sdev"      "loadings"  "center"    "scale"
[2] "n.obs"     "scores"    "call"
$class
[1] "princomp"
> round(pca$scores, digits=2)

```

	Comp.1	Comp.2	Comp.3	Comp.4	Comp.5	Comp.6		Comp.1	Comp.2	Comp.3	Comp.4	Comp.5	Comp.6
Courtellary	0.36	1.40	0.86	0.91	-0.63	-0.28	Payerne	-0.49	0.83	1.10	-0.78	-0.28	-0.37
Delemont	-1.63	1.03	-0.55	0.51	-0.65	-0.09	Paysd'enhaut	-0.94	-1.19	0.98	0.13	-0.85	-0.46
Franches-Mnt	-2.10	0.75	-0.47	1.50	-0.41	-0.09	Rolle	0.56	-1.61	0.35	-0.11	-0.19	-0.13
Moutier	-0.75	0.60	0.58	1.07	-0.23	-0.31	Vevey	1.96	0.60	0.02	-0.20	-0.08	0.25
Neuveville	0.38	0.45	0.63	0.25	-0.07	-0.77	Yverdon	0.12	0.41	0.91	-0.68	-0.55	0.09
Porrentruy	-1.37	2.29	-0.35	-0.31	-0.84	0.71	Conthey	-2.41	-2.04	-1.05	0.44	-0.13	0.02
Broye	-1.72	1.13	-0.43	-0.47	0.83	0.08	Entremont	-1.96	-0.73	-1.05	-0.73	-0.13	0.22
Glane	-2.18	1.76	-0.40	-0.25	0.81	-0.27	Herens	-2.59	-1.13	-0.84	-0.19	0.08	0.04
Gruyere	-1.52	0.62	-0.71	0.47	0.20	0.23	Martigwy	-1.52	-0.61	-0.98	-0.45	0.32	0.39
Sarine	-0.96	1.90	-0.66	0.00	0.30	0.03	Monthey	-2.10	-0.05	-0.66	0.23	-0.15	0.31
Veveyse	-2.01	1.54	-0.41	-0.29	0.63	0.12	St Maurice	-1.24	-1.15	-1.34	-0.34	-0.16	0.29
Aigle	0.81	-1.36	0.34	-0.08	0.44	-0.28	Sierre	-3.01	-1.23	-0.80	0.97	0.31	-0.65
Aubonne	-0.18	-0.88	0.83	-0.50	-0.19	-0.29	Sion	-1.08	-0.33	-1.24	0.52	0.43	-0.27
Avenches	0.24	0.49	0.81	-0.96	0.12	-0.38	Boudry	1.17	0.38	0.79	0.13	0.61	0.02
Cossonay	0.42	-1.07	0.93	-0.74	0.54	0.25	La Chauxdfnd	1.99	0.93	0.72	0.78	0.29	0.86
Echallens	-0.60	-0.30	0.86	-0.90	0.41	0.32	Le Locle	1.31	0.41	0.57	1.14	-0.03	0.06
Grandson	0.47	0.18	0.97	0.47	-0.32	0.03	Neuchatel	3.08	1.92	-0.34	-0.26	0.80	-0.46
Lausanne	2.78	0.64	-0.41	-0.09	-0.25	-0.24	Val de Ruz	-0.01	0.22	1.04	0.65	-0.27	-0.21
La Vallee	3.43	-2.08	-0.24	1.71	0.51	0.24	ValdeTravers	1.36	0.34	0.95	0.79	0.17	0.73
Lavaux	0.15	-0.66	0.78	-0.96	0.32	-0.27	V. De Geneve	5.66	0.56	-2.66	-0.32	-0.04	-0.44
Morges	0.71	-0.89	0.63	-0.22	0.50	-0.12	Rive Droite	1.86	-0.71	-1.64	-0.83	-0.94	-0.23
Moudon	-0.25	0.16	1.19	-0.79	-0.53	0.34	Rive Gauche	2.55	0.01	-1.66	-0.59	-0.70	0.38
Nyone	1.29	-1.26	0.14	-0.11	0.15	0.26							
Orbe	0.95	-1.87	0.60	0.14	0.07	0.42							
Oron	-0.97	-0.41	1.32	-0.64	-0.18	-0.14							

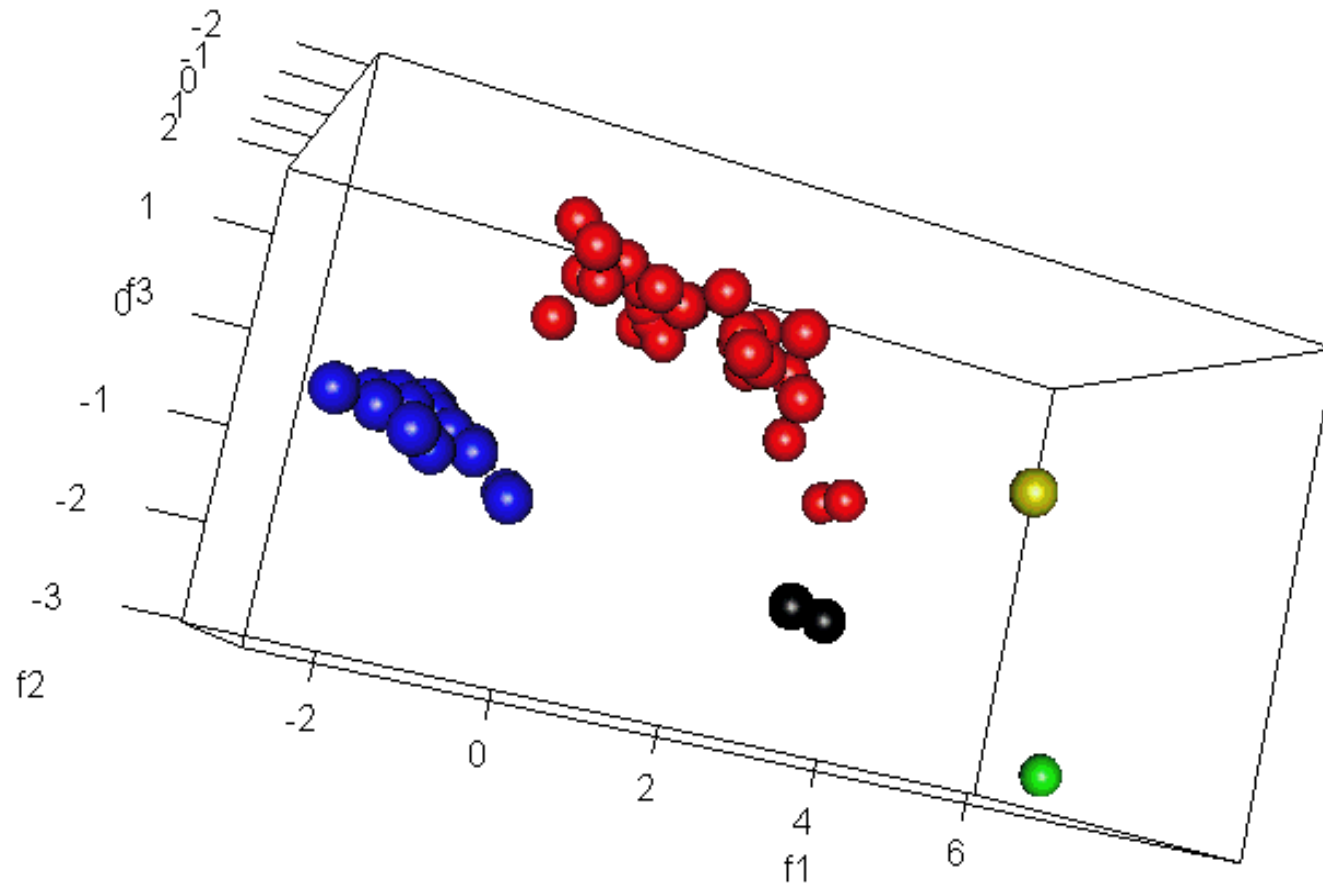
Balidazioa

```
> plot3d(pca$scores[,1],pca$scores[,2],pca$scores[,3],  
+        xlab="f1",ylab="f2",zlab="f3",  
+        aspect=sd(pca$scores[,1:3]),  
+        type = "s", size=1, col="black")
```



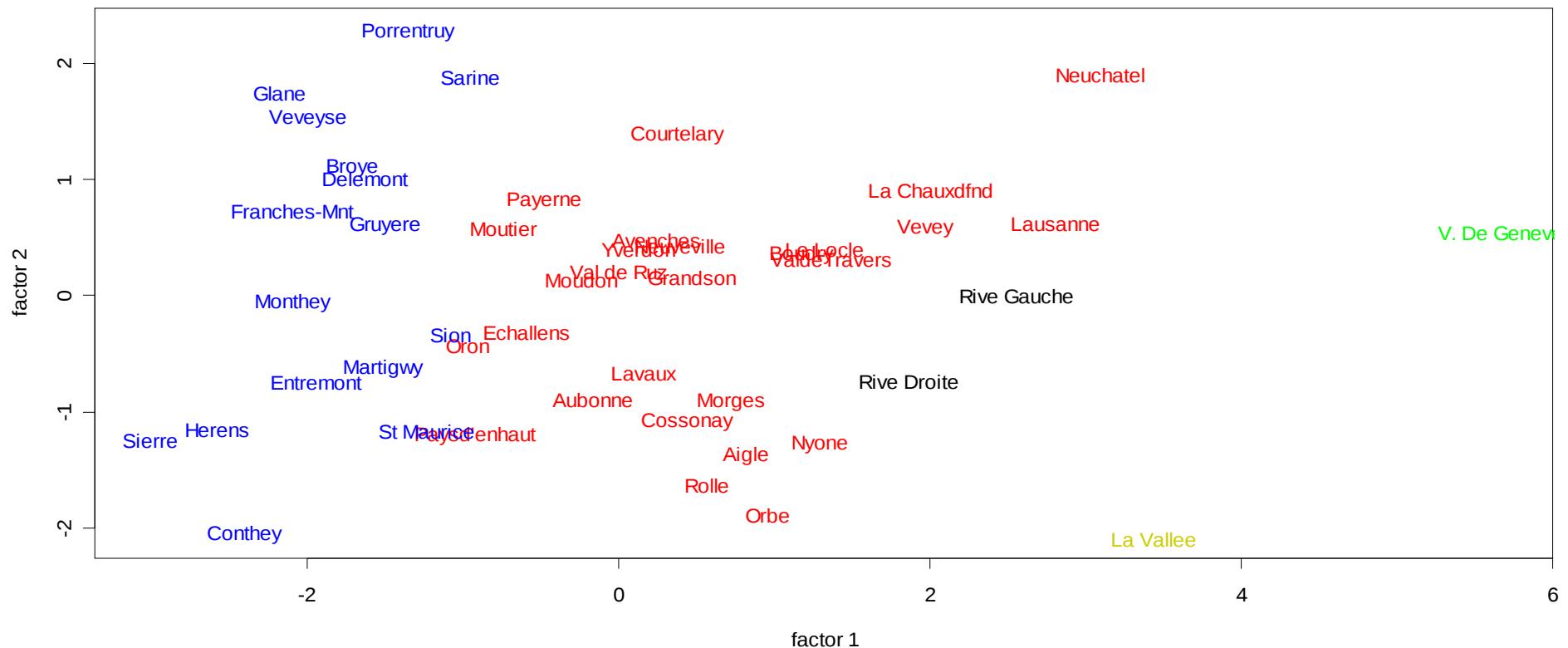
Balidazioa

```
> colores <- c("red", "blue", "yellow3", "green", "black")  
> plot3d(pca$scores[,1],pca$scores[,2],pca$scores[,3],  
+        xlab="f1",ylab="f2",zlab="f3",  
+        aspect=sd(pca$scores[,1:3]),  
+        type = "s", size=1, col=colores[cuthaverage])
```



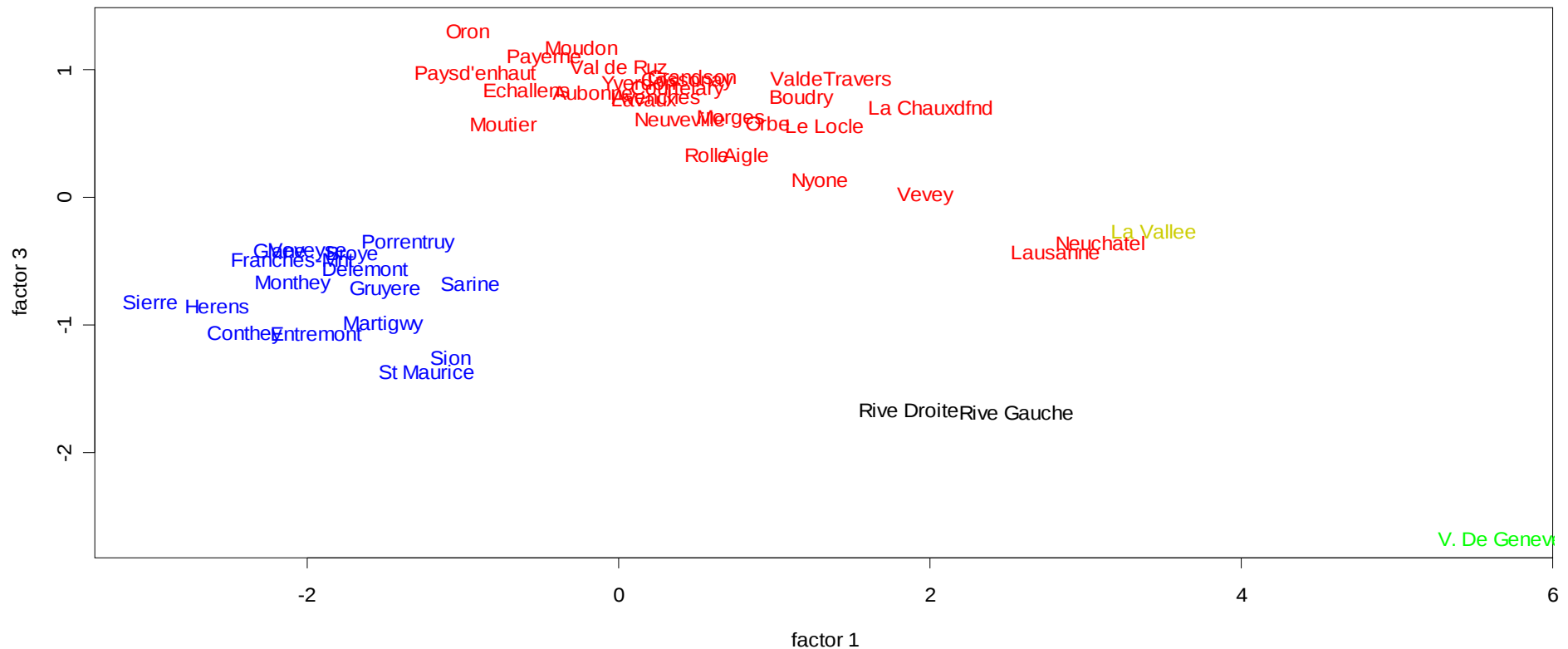
Balidazioa

```
> colores <- c("red", "blue", "green", "cornsilk", "black")
> plot(pca$scores[,1], pca$scores[,2], type="n",
       xlab="factor 1", ylab="factor 2", main="")
> text(pca$scores[,1], pca$scores[,2], labels=row.names(swiss),
       col=colores[cuthaverage])
```



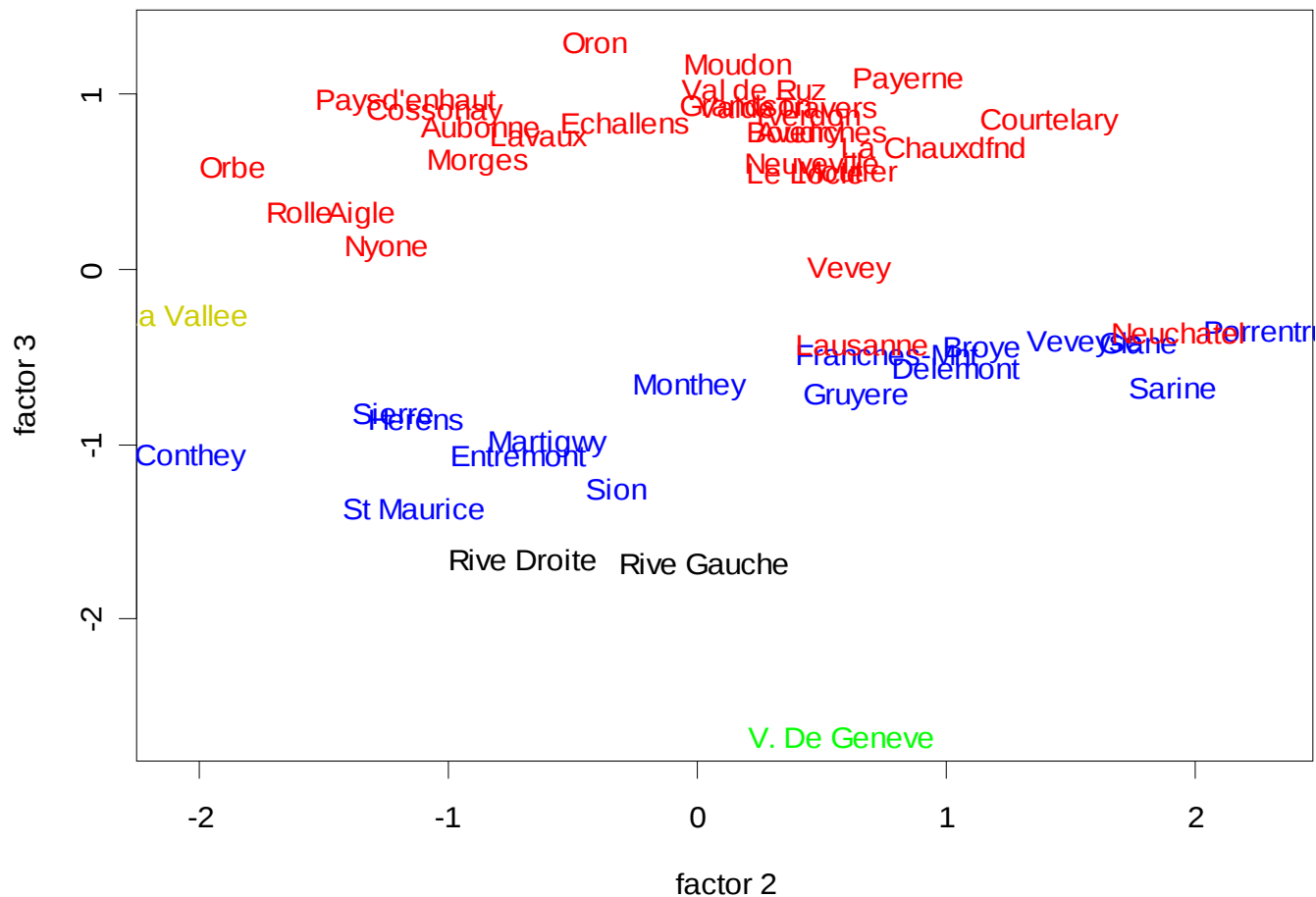
Balidazioa

```
> colores <- c("red", "blue", "green", "cornsilk", "black")
> plot(pca$scores[,1], pca$scores[,3], type="n",
       xlab="factor 1", ylab="factor 3", main="")
> text(pca$scores[,1], pca$scores[,3], labels=row.names(swiss),
       col=colores[cuthaverage])
```



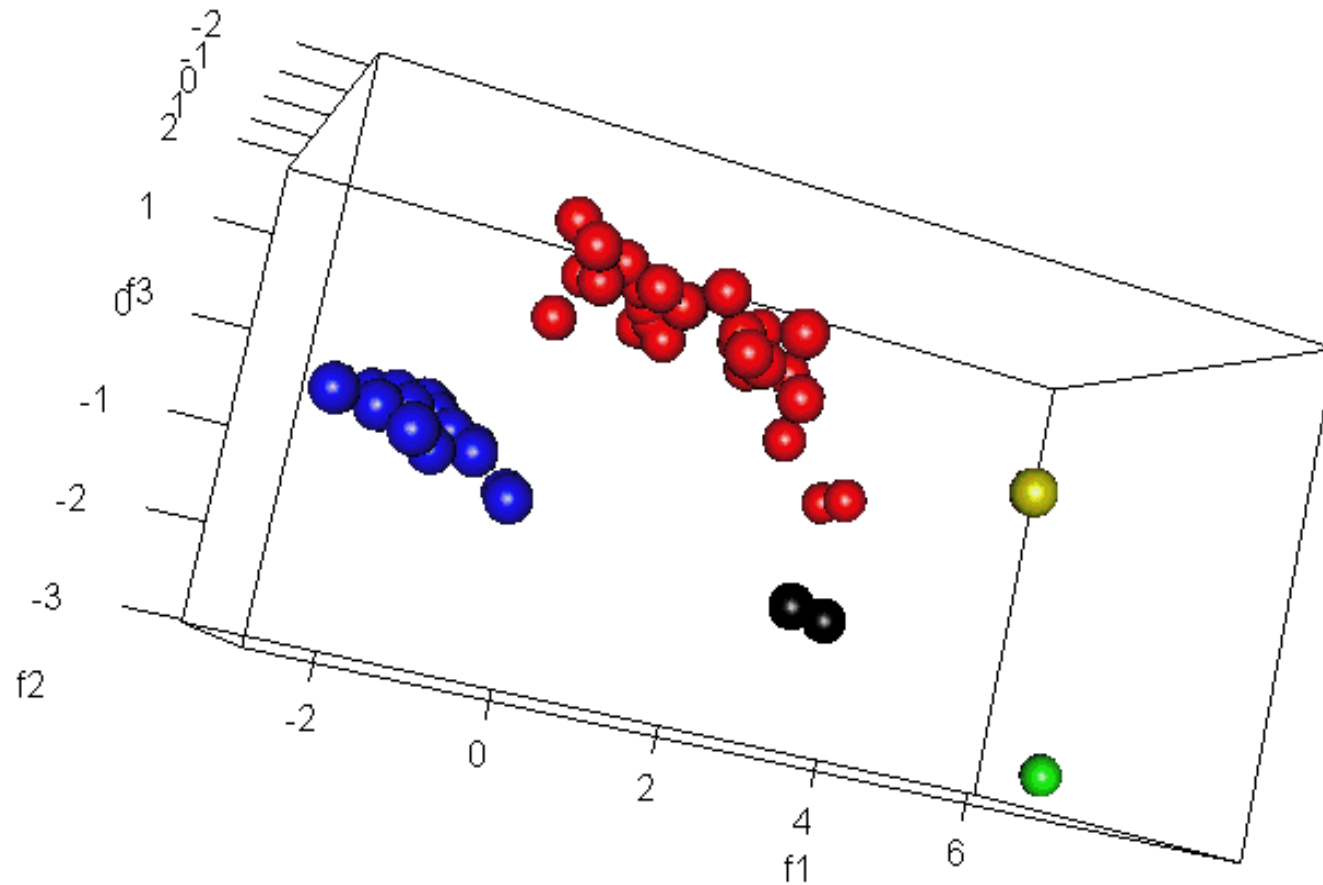
Balidazioa

```
> colores <- c("red", "blue", "green", "cornsilk", "black")
> plot(pca$scores[,2], pca$scores[,3], type="n",
       xlab="factor 2", ylab="factor 3", main="")
> text(pca$scores[,2], pca$scores[,3], labels=row.names(swiss),
       col=colores[cuthaverage])
```



Balidazioa

```
> colores <- c("red", "blue", "yellow3", "green", "black")  
> plot3d(pca$scores[,1],pca$scores[,2],pca$scores[,3],  
+        xlab="f1",ylab="f2",zlab="f3",  
+        aspect=sd(pca$scores[,1:3]),  
+        type = "s", size=1, col=colores[cuthaverage])
```



Interpretazioa

Datu-taularen lerroak eta zutabeak berrordenatu factor nagusiaren arabera.

```
> order(pca$scores[,1])
[1] 37 33 31 8 3 35 11 32 7 2 34 9 6 36 38 25
[17] 10 27 4 16 26 22 13 43 30 20 14 1 5 15 17 28
[27] 21 12 24 39 23 41 44 46 29 40 47 18 42 19 45
> order(pca$loadings[,1])
[1] 1 2 5 6 4 3
> cutav2 <- cutav[order(pca$scores[,1])]
> cutav2
[1] 2 2 2 2 2 2 2 2 2 2 2 2 2 2 1 2 1 1 1 1 1 1 1
[25] 1 1 1 1 1 1 1 1 1 1 1 1 1 1 5 1 1 5 1 1 3 4
> round(cor(swiss,pca$scores),digits=3)
```

	Comp.1	Comp.2	Comp.3	Comp.4	Comp.5	Comp.6
Fertility	-0.817	0.351	0.160	0.355	0.173	-0.164
Agriculture	-0.759	-0.449	-0.035	-0.426	0.170	-0.107
Examination	0.912	0.136	0.084	-0.036	0.368	0.078
Education	0.813	0.195	-0.490	-0.065	-0.032	-0.237
Catholic	-0.626	0.159	-0.743	0.066	0.083	0.140
Infant.Mortality	-0.268	0.884	0.147	-0.349	-0.047	0.026

Interpretazioa

Datu-taularen lerroak eta zutabeak berrordenatu factor nagusiaren arabera.

	Fertility	Agriculture	Catholic	Infant.Mortality	Education	Examination	cluster5
Sierre	92.2	84.6	99.46	16.3	3	3	2
Herens	77.3	89.7	100.00	18.3	2	5	2
Conthey	75.5	85.9	99.71	15.1	2	3	2
Glane	92.4	67.8	97.16	24.9	8	14	2
Franches-Mnt	92.5	39.7	93.40	20.2	5	5	2
Monthey	79.4	64.9	98.22	20.2	3	7	2
Veveyse	87.1	64.5	98.61	24.5	6	14	2
Entremont	69.3	84.9	99.68	19.8	6	7	2
Broye	83.8	70.2	92.85	23.6	7	16	2
Delemont	83.1	45.1	84.84	22.2	9	6	2
Martigwy	70.5	78.2	98.96	19.4	6	12	2
Gruyere	82.4	53.3	97.67	21.0	7	12	2
Porrentruy	76.1	35.3	90.57	26.6	7	9	2
St Maurice	65.0	75.9	99.06	17.8	9	9	2
Sion	79.3	63.1	96.83	18.1	13	13	2
Oron	72.5	71.2	2.40	21.0	1	12	1
Sarine	82.9	45.2	91.38	24.4	13	16	2

Interpretazioa

Datu-taularen lerroak eta zutabeak berrordenatu factor nagusiaren arabera.

	Fertility	Agriculture	Catholic	Infant.Mortality	Education	Examination	cluster5
Paysd'enhaut	72.0	63.5	2.56	18.0	3	6	1
Moutier	85.8	36.5	33.77	20.3	7	12	1
Echallens	68.3	72.6	24.20	21.2	2	18	1
Payerne	74.2	58.1	5.23	23.8	8	14	1
Moudon	65.0	55.1	4.52	22.4	3	14	1
Aubonne	66.9	67.5	2.27	19.1	7	14	1
Val de Ruz	77.6	37.6	4.97	20.0	7	15	1
Yverdon	65.4	49.5	6.10	22.5	8	15	1
Lavaux	65.1	73.0	2.84	20.0	9	19	1
Avenches	68.9	60.7	4.43	22.7	12	19	1
Courtelary	80.2	17.0	9.96	22.2	12	15	1
Neuveville	76.9	43.5	5.16	20.6	15	17	1
Cossonay	61.7	69.3	2.82	18.7	5	22	1
Grandson	71.7	34.0	3.30	20.0	8	17	1
Rolle	60.5	60.8	7.72	16.3	10	16	1
Morges	65.5	59.8	5.23	18.0	10	22	1
Aigle	64.1	62.0	8.52	16.5	12	21	1
Orbe	57.4	54.1	4.20	15.3	6	20	1
Boudry	70.4	38.4	5.62	20.3	12	26	1
Nyone	56.6	50.9	15.14	16.7	12	22	1
Le Locle	72.7	16.7	11.22	18.9	13	22	1
ValdeTravers	67.6	18.7	8.65	19.5	7	25	1
Rive Droite	44.7	46.6	50.43	18.2	29	16	5
Vevey	58.3	26.8	18.46	20.9	19	25	1
La Chauxdfnd	65.7	7.7	13.79	20.5	11	29	1
Rive Gauche	42.8	27.7	58.33	19.3	29	22	5
Lausanne	55.7	19.4	12.11	20.2	28	26	1
Neuchatel	64.4	17.6	16.92	23.0	32	35	1
La Vallee	54.3	15.2	2.15	10.8	20	31	3
V. De Geneve	35.0	1.2	42.34	18.0	53	37	4

Interpretazioa

Datu-taularen lerroak eta zutabeak berrordenatu factor nagusiaren arabera.

	Fertility	Agriculture	Catholic	Infant.Mortality	Education	Examination	cluster5
Sierre	92.2	84.6	99.46	16.3	3	3	2
Herens	77.3	89.7	100.00	18.3	2	5	2
Conthey	75.5	85.9	99.71	15.1	2	3	2
Glane	92.4	67.8	97.16	24.9	8	14	2
Franches-Mnt	92.5	39.7	93.40	20.2	5	5	2
Monthey	79.4	64.9	98.22	20.2	3	7	2
Veveyse	87.1	64.5	98.61	24.5	6	14	2
Entremont	69.3	84.9	99.68	19.8	6	7	2
Broye	83.8	70.2	92.85	23.6	7	16	2
Delemont	83.1	45.1	84.84	22.2	9	6	2
Martigwy	70.5	78.2	98.96	19.4	6	12	2
Gruyere	82.4	53.3	97.67	21.0	7	12	2
Porrentruy	76.1	35.3	90.57	26.6	7	9	2
St Maurice	65.0	75.9	99.06	17.8	9	9	2
Sion	79.3	63.1	96.83	18.1	13	13	2
Sarine	82.9	45.2	91.38	24.4	13	16	2

Interpretazioa

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	Fertility	Agriculture	Catholic	Infant.Mortality	Education	Examination	cluster5
Oron	72.5	71.2	2.40	21.0	1	12	1
Paysd'enhaut	72.0	63.5	2.56	18.0	3	6	1
Moutier	85.8	36.5	33.77	20.3	7	12	1
Echallens	68.3	72.6	24.20	21.2	2	18	1
Payerne	74.2	58.1	5.23	23.8	8	14	1
Moudon	65.0	55.1	4.52	22.4	3	14	1
Aubonne	66.9	67.5	2.27	19.1	7	14	1
Val de Ruz	77.6	37.6	4.97	20.0	7	15	1
Yverdon	65.4	49.5	6.10	22.5	8	15	1
Lavaux	65.1	73.0	2.84	20.0	9	19	1
Avenches	68.9	60.7	4.43	22.7	12	19	1
Courtelary	80.2	17.0	9.96	22.2	12	15	1
Neuveville	76.9	43.5	5.16	20.6	15	17	1
Cossonay	61.7	69.3	2.82	18.7	5	22	1
Grandson	71.7	34.0	3.30	20.0	8	17	1
Rolle	60.5	60.8	7.72	16.3	10	16	1
Morges	65.5	59.8	5.23	18.0	10	22	1
Aigle	64.1	62.0	8.52	16.5	12	21	1
Orbe	57.4	54.1	4.20	15.3	6	20	1
Boudry	70.4	38.4	5.62	20.3	12	26	1
Nyone	56.6	50.9	15.14	16.7	12	22	1
Le Locle	72.7	16.7	11.22	18.9	13	22	1
ValdeTravers	67.6	18.7	8.65	19.5	7	25	1
Vevey	58.3	26.8	18.46	20.9	19	25	1
La Chauxdfnd	65.7	7.7	13.79	20.5	11	29	1
Lausanne	55.7	19.4	12.11	20.2	28	26	1
Neuchatel	64.4	17.6	16.92	23.0	32	35	1
Rive Droite	44.7	46.6	50.43	18.2	29	16	5
Rive Gauche	42.8	27.7	58.33	19.3	29	22	5
La Vallee	54.3	15.2	2.15	10.8	20	31	3
V. De Geneve	35.0	1.2	42.34	18.0	53	37	4