

# OPTIMIZATION OF DAM CONSTRUCTION COSTS USING GENETIC ALGORITHMS

Marcos M. Silvoso, Eduardo M. R. Fairbairn, Member ASCE Romildo D. T. Filho, Nelson F. F. Ebecken, and José L. D. Alves

### ABSTRACT

This paper presents a procedure to optimize the construction of mass concrete structures using genetic algorithms. Due to cement hydration, thermal and shrinkage strains take place in concrete at early ages and, if they are restrained, tensile stresses develop in the concrete structure. As a consequence, cracks may appear if the magnitude of the generated stresses reaches the concrete tensile strength. In this study, transient hydration and thermal and stress fields were calculated using a coupled thermo-chemo-mechanical model implemented in a 3D finite element code. The optimization criterion is construction cost and the decision variables are material types, characterized by their mechanical and hydration properties; placing temperature, the height of lifts and time intervals between lifts. The constraint imposed on the decision variables is the early age cracking of the structure. A dynamic penalty scheme that allowed a certain level of cracking for the preliminary generations proved to be efficient in driving the genetic algorithm to an optimal solution. To show the potential of the proposed methodology, the construction phase of a small hydropower plant dam was optimized. The results indicated that the procedure can be successfully used in the design of massive concrete structures.

**Keywords:** mass concrete, hydration reaction, autogenous shrinkage, thermal stress, early age cracking, numerical modeling, optimization, genetic algorithms

# INTRODUCTION

Massive concrete structures such as dams, foundation slabs and bridge decks may be subject to early age cracking due to thermal stresses and autogenous shrinkage-induced stresses. From the engineering point of view, if cracking must be avoided, several measures can be undertaken to reduce the early effects of the hydration reaction, such as:

- 1. choosing a material composition that gives lower rates of hydration and/or limiting the autogenous shrinkage potential;
- 2. controlling lift thickness and the time intervals between lifts to allow heat to dissipate;
- 3. reducing the placing temperature of concrete.

Because of the large volume of concrete and the logistics involved in construction of massive structures, an accurate and feasible procedure that could be used consistently to optimize the variables intervening in mass concrete construction would be very helpful to both the designer and the contractor.

Dept. of Civil Eng., COPPE/UFRJ, Rio de Janeiro, RJ 21945-970. e-mail: silvoso@labest.coc.ufrj.br; eduardo@coc.ufrj.br; toledo@labest.coc.ufrj.br; jalves@lamce.ufrj.br

An extensive survey on Cost Optimization of Concrete Structures (Sarma, K. C. and Adeli, 1998) showed that the great majority of papers dealing with structural optimization concerned weight minimization. Of the small number of papers on cost optimization, most were dedicated to simple elements such as beams and girders, and very few used costs functions that considered the costs of placement and construction. Since 1998, the panorama has not essentially changed and no reference on the optimization of mass concrete structures was found, although mass concrete cost optimization can result in substantial savings.

This paper presents a procedure for optimizing the construction phase of mass concrete structures. This is a cost optimization problem that involves the cost of the raw material and construction costs associated with placing, cooling, formwork, lift height and time intervals.

The procedure presented in this paper systematizes the cost optimization problem of mass concrete construction using:

- 1. a coupled thermo-chemo-mechanical model (Ulm & Coussy, 1995, 1996) implemented in a
  - 3D FEM code (Silvoso et al, 2001) to simulate the effects of the hydration reaction;
- 2. a genetic algorithm procedure (Castro, 2001) to optimize construction costs.

The potential of the the procedure presented in this paper is demonstrated through the optimization of the construction phase of a concrete dam for a small hydropower plant. The results indicate that the proposed procedure is both accurate and feasible and should become a useful tool for optimizing mass concrete construction.

### THE THERMO-CHEMO-MECHANICAL MODEL

For simulating the evolution of the hydration reaction and its effects we used the well-known model of Ulm and Coussy (1995,1996). The general concepts of this model are as follows:

- The model considers concrete as a reactive porous media composed by a solid skeleton of anhydrous cement grains and CSH hydrates and porous that may be filled either by air or by water.
- The thermo-activation and the exothermic behavior of the hydration reaction are taken into account by the model.
- The evolution of the hydration reaction is given by the following equation:

$$\frac{dm}{dt} = \frac{d\xi}{dt} m_{\infty} = \frac{1}{\eta(\xi)} A(\xi) \exp\left(-\frac{E_a}{RT}\right)$$
(1)

where:

dm/dt represents the variation of mass of the skeleton;

 $0 < \xi < 1$  is the hydration degree. It can be understood as the relation between the mass of the skeleton at a time t normalized by the mass of the skeleton at the end of the hydration, i.e.,  $\xi(t) = m(t)/m_{\infty}$ ;

 $\eta(\xi)$  is a viscosity term corresponding to the growing of the physical barrier of CSH which tends to isolate the cement grain from the free water. It depends on the state of the hydration reaction;

 $A(\xi)$  is the affinity of the chemical reaction (the thermodynamical force associated to the rate of hydrates formation);

 $E_a$  is the apparent thermal activation energy considered constant with relation to the hydration degree;

R is the universal constant of gases, and T is the temperature in Kelvin.

• The equation corresponding to the transient thermal and hydration fields, considering the thermo-chemical coupling is given by:

$$C_{\varepsilon}\dot{T} = Q + L\dot{\xi} + k\nabla^2 T \tag{2}$$

Here, the standard form for the evolution of the thermal fields  $C_{\varepsilon}\dot{T} = Q + k\nabla^2 T$  can be recognized, with  $C_{\varepsilon}$  the specific heat and k the thermal conductivity. The term  $L\dot{\xi}$  represents the heat generated by the exothermic reaction, L being the latent heat of hydration determined by means of experimental tests. Equation (2) can have a step-by-step solution if the values of  $\dot{\xi}$  (and consequently the values of  $\xi$ ) are determined, for each time-step by means of equation (1). For this aim it is necessary to know the values of the normalized affinity  $\tilde{A}(\xi) = A(\xi)/(m_{\infty}\eta(\xi))$ . This may be experimentally obtained by means of adiabatic or quasi-adiabatic tests (Ulm & Coussy, 1995,1996).

• The chemo-mechanical couplings may be represented by the following equation:

$$d\boldsymbol{\varepsilon} = \mathbf{C}(\boldsymbol{\xi})^{-1} : d\boldsymbol{\sigma} + \mathbf{a}dT + \mathbf{\overline{b}}d\boldsymbol{\xi}$$
(3)

In this equation  $\boldsymbol{\varepsilon}$  is the strain tensor,  $\mathbf{C}(\boldsymbol{\xi})$  is the elastic compliance tensor dependent on the hydration degree,  $\boldsymbol{\sigma}$  is the stress tensor and ,  $\mathbf{a}$  is the tensor of the thermal deformation coefficients and  $\mathbf{\overline{b}}$  is the tensor of the chemical coefficients relating the hydration rate to the autogenous shrinkage. In this paper only the Young's modulus is considered as depending on the degree of hydration, the Poisson's ratio v being constant, we can write:  $\mathbf{C}(\boldsymbol{\xi}) = \mathbf{C}(E(\boldsymbol{\xi}))$ .

• The chemo-plastic coupling considers that the hardening forces  $\zeta$ , which are associated with the sizes of the yield surfaces in the stress space, depend on both the plastic hardening/softening variables  $\chi$  and the evolution of the hydration reaction  $\xi$ , reading, formally,  $\zeta(\chi, \xi)$ . Two yield surfaces are employed, namely, *Drucker-Prager* when the stress state is of the compressive type, and *tension-cut-off* when the stress state is of the framework of associative plasticity, one can write the flow and the hardening rules as follows:

$$d\boldsymbol{\varepsilon}^{\mathbf{p}} = \sum_{s \in Jact} d\lambda_s \frac{\partial f_s}{\partial \boldsymbol{\sigma}} \quad ; \quad d\boldsymbol{\chi}_1 = d\lambda_1 \frac{\partial f_1}{\partial \zeta_1} \quad ; \quad d\boldsymbol{\chi}_2 = d\lambda_2 \frac{\partial f_2}{\partial \zeta_2} \tag{4}$$

where *Jact* is the set of active surfaces, i.e. the surfaces for which  $f_s(\sigma, \zeta)=0$ ; and  $d\lambda_s$  are plastic multipliers, defined as in standard plasticity. Given that only the Drucker-Prager criterion allows for hardening, the two criteria used in this paper, represented by the functions  $f_1$  and  $f_2$ , can be written as:

$$f_1 = \alpha (I_1 + 3\zeta_1(\chi_1, \xi)) + J_2^{1/2} \le 0 \quad ; \quad \alpha = \frac{1}{\sqrt{3}} \frac{\kappa - 1}{2\kappa - 1}$$
(5)

$$f_2 = I_1 + 3\zeta_2(\xi) \le 0$$
 (6)

where  $I_1 = tr(\sigma)$  and  $J_2 = (s : s)/2$  are, respectively, the first invariant of the stress tensor and the second invariant of the deviator stress tensor and  $\kappa = f_b(\xi)/f_c(\xi)$  is the ratio between the biaxial and the uniaxial compressive strengths, considered to be constant ( $\kappa$ =1.16) with hydration. The evolution laws for the compressive and tensile strengths, Young's modulus, autogenous shrinkage, and plastic variables are described in detail by Silvoso (2003) and Fairbairn et al (2003).

This model has been implemented in a 3D FEM code with vectorization facilities allowing a better computational performance. The code uses linear tetrahedral elements and the non-linear system of equations is solved by using a Newton-Raphson incremental iterative technique. An element-by-element strategy solution with preconditioned conjugate gradient solver with diagonal preconditioner of the elements matrices (Coutinho et al, 2001). The code has two modules. The first module computes the transient thermal and hydration fields. The second module calculates the stresses and strain fields for each time step.

For simulating the construction process each element is associated to a construction phase and consequently to the activation time of this section of the structure. When the activation time is reached, the corresponding finite elements are activated and the boundary conditions are updated. It should be pointed out that the data structure of the computer code and the element-by-element solution improve the efficiency of the numerical analysis and make easier the simulation of the construction phases.

# GENETIC ALGORITHMS FOR THE OPTIMIZATION OF MASS CONCRETE CONSTRUCTION

The design variables considered for the optimization problem of thermal cracking for massive concrete structures are:

- The type of concrete (*tc*). The choice of the material is directly related to the evolution of the hydration reaction (heat generation, autogenous shrinkage) and also to the development of the properties of the skeleton such as the tensile strength and the Young's modulus. As an example, use of a pozzolan as cement replacement further delays and reduces heat generation. The type of concrete is a discrete variable, reading *tc* ∈ {1,2,... N<sub>tc</sub>}, where N<sub>tc</sub> is the number of types of concrete.
- The placing temperature (*pt*). Since the hydration reaction is thermo-activated, cooling the concrete will contribute to slow down the chemical reaction and to reduce the cracking risk. It is evident that in spite of the favorable effects, precooling the concrete will increase the construction cost. The variable can then be defined by taking its values from the discrete set *pt* ∈ {*pt*<sub>1</sub>=*ptmin*, *pt*<sub>2</sub>,...,*pt*<sub>Npt</sub>=*pt*<sub>max</sub>=*Tenv*}, where *pt*<sub>min</sub> is the minimum feasible placing temperature, *Npt* is the number of placing temperatures, and *pt*<sub>max</sub>=*Tenv* is the maximum placing temperature, which corresponds to placing concrete at the ambient temperature (*Tenv*).
- The height of lifts (*hl*). This variable influences the dissipation of the heat generated by the hydration reaction. The shallower the lift, the more heat of hydration will escape before the next lift is placed and the maximum temperature reached will be lower. On the other hand, the higher the lifts, the fewer the horizontal construction joints and the lower the construction costs. Since the values of *hl* are limited by a minimum and maximum feasible height, we can write  $hl \in \{hl_1=hl_{min}, hl_2,..., hl_{Nhl}=hl_{max}\}$ .
- The placing frequency (*pf*). The time period after the construction of one lift, before placing the next lift, allows heat to dissipate. If this time period increases, the maximum temperature reached in the lift will be lower but the construction cost will increase. It is

also, generally, a discrete variable, since the construction frequency is not established on the basis of fractions of days. It is limited by a minimum and maximum feasible placing frequency, as follows  $pf \in \{pf_i = pf_{min}, pf_2, ..., pf_{Npf} = pf_{max}\}$ , where Npf is the number of placing frequencies,  $pf_{min}$  and  $pf_{max}$  being, respectively, the minimum and the maximum placing frequencies as established by the contractor.

The vector of variables, designated by x, can then be defined as:

$$\mathbf{x}^{1} = \{x_{1}, x_{2}, x_{3}, x_{4}\} = \{tc, pt, hl, pf\}$$
(7)

Since the geometry of the structure is given, the cost function is established for a fixed total volume of concrete  $V_{Con,tot}$ , as follows:

$$C/V_{Con,tot} = c = (c_{Fixed} + c_{RM}(tc) + c_{CC}(pt) + c_{PO}(hl, pf))$$
(8)

where:C is the total cost; c is the total unit cost; cFixed represents all the unit costs that do not vary with *hl*, *pf*, *tc* and *pt*, such as the cost of plant purchase, installation and other constant costs of the activities related to  $pf_{min}$ ;  $c_{RM}(tc)$  is the unit cost of the raw material, which is a function of the type of concrete;  $c_{CC}(pt)$  is the unit cost of concrete cooling, which is a function of the placing temperature; and  $c_{PO}(hl, pf)$  is the unit cost of plant operation, which varies with the total construction time, which in turn depends on the height of the lifts and the placing frequency. This cost includes leasing and operation of equipment, concrete treatment, consolidation and placing (with the exception of cooling) and so on.

The unit costs are expressed as proportions that indicate their weighting in the total cost. These weights are defined by the coefficients  $\omega$  as follows:

$$c_{Fixed} = \omega_{Fixed}.c; \quad c_{RM} = \omega_{RM}.c; \quad c_{CC} = \omega_{CC}.c; \quad c_{PO} \cong \omega_{PO}.c; \quad \omega_{Fixed} + \omega_{RM} + \omega_{CC} + \omega_{PO} = 1$$
(9)

The values of these weights depend on factors such as the type of construction, local conditions, variations in the unit costs, etc. However, regardless of the fuzziness of these unit costs, it is always possible to estimate them when a project is being planned.

If  $\tilde{c}(x)$  is defined as the normalized variable unit cost:

$$\widetilde{c}(\mathbf{x}) = \frac{c_{RM}(tc) + c_{CC}(pt) + c_{PO}(hl, pf)}{c_{RM,\max} + c_{CC,\max} + c_{PO,\max}}, \quad \widetilde{c}(\mathbf{x}) \in [\widetilde{c}_{\min}, 1]$$
(10)

where  $\tilde{c}_{\min}$  is the minimum normalized cost, obtained for  $c_{RM}=c_{RM,\min}$ ,  $c_{CC}=c_{CC,\min}$  and  $c_{PO}=c_{PO,\min}$ , the objective function can be expressed as:

$$f(\mathbf{x}) = \widetilde{c}(\mathbf{x}) \tag{11}$$

We define  $ECr(x) \in [0,1]$  as a measure of the extent of cracking, taken in this paper to be the plastified finite elements volume ratio, such that ECr(x)=0 for an uncracked structure, the optimization problem can be stated as:

- Minimize f(x)
- Under the constraint defined as ECr(x)=0

The constraint, i.e., the rejection of cracked structures, is handled by a penalty scheme. A fitness function, set out below, is therefore introduced:

- F=f(x) for an uncracked structure
- $F=f(\mathbf{x})+P(ECr(\mathbf{x}),t_g)$  for a cracked structure

The penalty  $P(ECr(\mathbf{x}), t_g)$  is a function of both  $ECr(\mathbf{x})$ , and  $t_g$ , which represents the generation.

The penalty function is bilinear such that, in the first half of the generations (i.e., until  $t_g = Ng/2$ , Ng being the number of generations) the individuals that present cracking under a certain cracking threshold  $(ECr_{\text{lim}}(t_g))$ , are not fully penalized. For generations after Ng/2, the penalty becomes static and all cracked structures are penalized by the same value  $P(\mathbf{x}, t_g) = 1 - \tilde{c}_{\min}$ , the penalty function can be written as follows:

$$P(ECr(\mathbf{x}), t_g) = \begin{cases} 1 - \widetilde{c}_{\min} & \text{, if } ECr(\mathbf{x}) > ECr_{\lim}(t_g) \\ ECr(\mathbf{x}) \frac{1 - \widetilde{c}_{\min}}{ECr_{\lim}(t_g)} & \text{, if } ECr(\mathbf{x}) \le ECr_{\lim}(t_g) \end{cases}$$
(12)

$$ECr_{lim}(t_g) = \begin{cases} ECr_{lim,0} - t_g \frac{ECr_{lim,0}}{Ng/2} , \text{ if } 0 \le t \le Ng/2 \\ 0 , \text{ if } Ng/2 \le t_g \le Ng \end{cases}$$
(13)

A general sketch of the genetic algorithm used in this paper is presented in the form of a flowchart in Figure 1. The algorithm is terminated when the maximum number of generations (Ng) is achieved.



Figure 1 – Flowchart of the Genetic Algorithm

#### Example

We analyze a typical project of a small hydropower concrete dam that is 10m high and its total concrete volume is 3000 m<sup>3</sup>. The main geometric characteristics and the finite element mesh used for the following simulations are shown in Figure 2.

The design variables, as defined in above, are:

<u>Type of Concrete</u> (tc) – Eight types of concretes were used, all taken from a dam concrete laboratory data bank (Andrade, 1997). The variable tc is thus defined as  $tc \in \{1, 2, ..., 8\}$ . The composition of these types of concretes is given in Table 1 and their thermo-chemo-mechanical parameters are given in Table 2. The activation energy and the parameter  $\xi_0$  were considered to be constant for all the materials, reading, respectively,  $E_a/R(1,...,8)=4000/^{O}$ K and  $\xi_0(1,...,8)=0.1$ . The curves of the adiabatic temperature rise are given in Figure 3.

<u>Placing temperature (*pt*)</u>: This variable is defined as  $pt(^{\circ}C) \in \{10,11,...,24,25\}$ , where  $10^{\circ}C$  is the minimum cooling temperature available for the present application, and  $25^{\circ}C$  is the average ambient temperature.

<u>Height of the lifts</u> (*hl*): The values used in the present application are  $hl(m) \in \{0.5, 0.75, 1.0, 1.25, 1.5, 1.75, 2.0, 2.5\}$ , with each value producing a corresponding number of lifts  $NL \in \{20, 14, 10, 8, 7, 6, 5, 4\}$ .

<u>Placing frequency</u> (*pf*): This variable was given a range of values  $pf(days) \in \{6,7,...,20,21\}$ . A minimum value of 6 days was chosen because it was assumed that 5 days is the minimum period of time needed to cure the concrete and prepare the horizontal joint and the formwork for the new lift. Given the limit values adopted for *hl* and *pf*, the construction time can vary between 24 and 405 days.



Figure 2 – Geometric characteristics of the dam and finite element mesh

tc	$B_1$ (cement)	$B_2(\text{fly ash})$	$B_3(slag)$	W	$G_1(fine)$	$G_2(\text{coarse})$
1	139		170	165	626	1318
2	338	57		190	664	1057
3	299	50		187	700	1068
4	336			185	745	1090
5	335			160	603	1197
6	323			178	852	1084
7	127		237	163	655	1165
8	239		80	171	621	1303

Table 1 – Composition of the eight types of concretes (kg/m3)

tc	$C_{\varepsilon}(J.kg/K)$	<i>k</i> W/(m.K)	α (10-6)	$f_{c,\infty}(\mathrm{MPa})$	$E_{\infty}$ (MPa)	$\mathcal{E}^{sh}_{\infty}(10^{-6})$
1	1017	2.65	13.02	29.9	21.7	23.46
2	1109	2.64	10.78	28.9	30.6	21.09
3	1134	2.64	10.37	24.8	25.9	11.37
4	1084	2.64	10.62	30.2	26.0	24.17
5	1059	2.64	12.03	27.3	22.4	17.30
6	1092	2.24	9.93	23.9	23.2	10.05
7	1063	2.26	12.58	25.4	24.0	12.79
8	1050	2.49	12.09	25.2	17.1	12.32

Table 2 – Thermo-chemo-mechanical parameters for the eight types of concretes



Figure 3 – Adiabatic temperature rise for the eight types of concretes

In this example, the weighting of the cost components, as defined by formulas (9), was assumed to be, on average  $\omega_{Fixed}=35\%$ ;  $\omega_{RM}=30\%$ ;  $\omega_{CC}=5\%$ ; and  $\omega_{PO}=30\%$ . The unit costs  $c_{RM}$ (tc), for the several types of concrete, are calculated using using the quantities of raw material shown in Table 1 and assuming that the unit cost of the components is that given in Table 3. The results are shown in Table 4, from which it can be deduced that the minimum and maximum costs are  $c_{RM,min}=24.08 \text{ US}\text{s/m}^3$ , and  $c_{RM,max}=34.60 \text{ US}\text{s/m}^3$ . The values used for  $c_{CC}(pt)$  are given in Table 5, where it can be seen that  $c_{CC,min}=0.0 \text{ US}\text{s/m}^3$ , and  $c_{RC,max}=21.88 \text{ US}\text{s/m}^3$ .

Table 3 – Estimated unitar	y costs for the components o	of concrete (US\$/kg)
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Component	Unitary Cost
Cement	0.077
Fly ash	0.033
Ground granulated blast furnace slag	0.033
Aggregate	0.0077
Water	0.0077

tc	$c_{RM}$	tc	$c_{RM}$
1	24.08	5	34.60
2	34.93	6	33.11
3	30.88	7	26.73
4	34.51	8	27.96

Table 4 – Unitary costs of the concretes (US\$/m3)

pt (°C)	$C_{CC}$						
10	21.88	14	12.81	18	6.88	22	1.88
11	18.75	15	11.25	19	5.63	23	0.94
12	16.25	16	9.38	20	4.38	24	0.31
13	14.38	17	8.13	21	3.13	25	0.00

Table 5 – Unitary costs for cooling concrete (US\$/m<sup>3</sup>)

The unit cost  $c_{PO}(ct(hl,pf))$  is calculated based on the assumption that the relationship between the cost of plant operation and construction time is linear. The minimum and maximum costs are  $c_{PO,min}$ = 11.2 US\$/m<sup>3</sup>, and  $c_{PO,max}$ = 189.00US\$/ m<sup>3</sup>. With the values presented above,  $\tilde{c}_{min}$  is determined to be 0.144, using formula (10). The first cracking threshold for the penalty function was established as  $ECr_{lim,0}$ =0.03. This value, which was chosen on the basis of experience, indicates that first generation individuals with cracking of less than 3% are not discarded from the evolutionary process. If the cracking exceeds the threshold, the finite element calculation for the individual is terminated, because penalty applied is the same for  $ECr_{lim} < ECr \le 100\%$ . The control parameters of the genetic algorithm set at N = 50 individuals, Nt = 5 individuals, Pc=90% and Pm=5%. The algorithm terminates when the maximum number of generations is reached (Ng=50). Figure 4 shows the evolution of the fittest individual. From this Figure we can conclude that the use of a penalty function that allows for a certain amount of cracking in the first generations was effective in avoiding stagnation in the evolutionary process. The analysis showed that the minimum cost is attained with a construction scheme described by the following optimal values: tc=8; pt=19°C; hl=1.25m; pf=6 days (ct=48 days).



Figure 4 – Evolution of fitness of the best individual



Some results obtained for this optimal set are given in Figure 5, (temperature fields for several construction steps), and Figure 6 (principal stress fields for several construction steps).

Figure 5 – Temperature fields (days) ((a) 2, (b) 14, (c) 26 , (d) 38, (e) 45, (f) 55



Figure 6 - Principal stress fields (days) (a) 2, (b) 14, (c) 26, (d) 38, (e) 45, (f) 55

# **CONCLUDING REMARKS**

This paper presents a new procedure that can help the engineer in deciding on the construction process and in choosing the composition of mass concrete structures. The procedure combines an advanced thermo-chemo-mechanical model and optimization techniques based on genetic algorithms. The procedure was applied to optimize the hypothetical construction of a small concrete dam and the results indicated that the use of the cost optimization procedure can result in substantial savings.

The procedure can readily be applied to the actual design of massive structures in which early age cracking is a predominant design constraint. The principal application of the procedure would therefore be in the optimization of concrete dams, massive foundation slabs and bridge decks and other structures made of high performance concretes for which autogenous shrinkage coupled with thermal stresses is a major concern.

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