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LOOP ANALYSIS FOR QUALITATIVE SIMULATION

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ABSTRACT. In this work we expound the use of Qualitative Simulation -QS- on the IS-LM macroeconomic model. Each one of the submodels can be effectively simulated by QS algorithms. However, in order to uniquely identify all the qualitative aspects of this second order system, we apply the Loop Analysis technique. The perspective is that, by establishing some requirements at the modeling stage, we can structure the modeling process and apply some more classical qualitative tools, specifically Loop Analysis.

I. INTRODUCTION

Qualitative reasoning and simulation techniques are being applied mainly to physical systems and explanation of mechanisms [7]. We think that these techniques can be successfully applied in the area of socioeconomic systems provided that a structured modeling process is laid down, as is the practice with numerical simulations. They would also help to explain the internal mechanisms of these areas and, from an academic perspective, would help the teaching of models without referencing to specific numerical cases.

From a modeling point of view, Qualitative Simulation -QS- can be enriched by using other modeling paradigms. Concepts that can be explicitly included are causality and structured model development. Some approaches that appear to be clearly valid are the use of causal diagrams, the System Dynamics method [3], and model representation by means of Loop Analysis -LA-.

II. LOOP ANALYSIS

The ambiguities of some results given by QS can be overcome by using filters or other methods [11]. Berleant showed how the actual results of a QS can be "interpreted" as an eventual approach to equilibrium in a second order system [1]. Instead of interpreting some results, we can grasp the global behavior of a system by specifying the model according to a signed digraph and applying the classical techniques of LA [9]

LA is a technique that derives the qualitative behavior of signed digraph models by analyzing the qualitative characteristics expressed in the signed digraph. Although not presented as LA, the usefulness of feedback analysis is also suggested in [6], as the use of the "Qualitative System properties".

LA calculations are carried out in terms of feedback loops. Every variable of a model is explicitly stated in a causal diagram, whilst the loop diagram is a more "compact" form of causal diagram.

The feedback level is the number of variables that are part of the feedback path. The feedback levels in any system will range from one to the total number of variables in the system ($F_0 = -1$). In general, feedback for any loop model at level k is found by determining all the loops of length k or product of disjunct loops - those that have no common variables - which have a combined length of k, and then adding them together

$$F_k = \sum_{m=1}^{k} (-1)^{m+1} L(m,k)$$

where $L(m,k)$ stands for m disjunct loops with k elements.

• *Stability*. Stability criteria based on the loop model notation are as follows:

1) feedback at all levels must be negative: $F_i < 0 \;\; \forall i$

2) the second condition relates the negative feedback of long loops to the negative feedback of short loops. It states that negative feedback at high levels cannot be too strong in comparison to lower levels. For three and four variables this criterion is expressed as $F_1F_2 + F_3 > 0$. For five variables the second condition for stability is $-(F_1F_2 + F_3) F_3 + (F_1F_4 + F_5) F_1 > 0$. For more variables the formula in terms of feedbacks can be correlated with the Routh-Hourwitz criterion, as explained in [9, Ch. 6]. The equivalence of the graph to the algebraic structure has already been developed in the context of specific analysis [5].

• *Qualitative Predictions*. The change in the equilibrium value of variable X_j due to a change in parameter c in one or more variables in the system network is denoted by $(\delta X_j^*/\delta c)$, where * identifies equilibrium value. The change in variable X_j due to a change in parameter c is calculated by the formula:

$$\frac{\delta X_j^*}{\delta c} = \frac{\sum_{i,k} (\frac{\delta f_i}{\delta c}) (p_{ji}^{(k)}) (F_{n-k}^{(comp)})}{F_n}$$

The formula reads, take all the functions which include the parameter being changed $(\delta f_i/\delta c)$, trace each possible path $(p_{ji}^{(k)})$ to the *j*th variable (X_j) whose equilibrium value is being calculated, multiply each path by the appropriate complementary feedback $(F_{n-k}^{(comp)})$. Sum $\Sigma_{i,k}$ for all functions and paths, and divide it by the overall feedback of the entire system of n variables (F_n). Complementary feedback is that related to those variable subsystems and their interconnections which are not included in the open path $p_{ji}^{(k)}$.

• *Limitations*: [2] gives an example of how stability properties derived by LA of a linearized system of a nonlinear system are, in general, not valid. Puccia [9] asserts that for nonequilibrium systems that are linearizable, LA predictions apply to average values instead of equilibrium values.

III. Example: The IS-LM Macroeconomic Model

A. The IS-LM Model.

The IS-LM is part of the keynesian model to determine the price level and income of an economy [4]. This model specifically deals with the goods and money sectors, and is one of the simplest macroeconomic models. Moreover, this model can be modeled as a causal diagram, loop diagram and QS. See Figure 1.

In this model, the goods and the money sectors determine the interest rate and production level through the intersection of IS and LM schedules.

The set of variables we use in the model are

Y: Output produced, level of income of an economy
C: Consumption or demand for goods
I: Investment spending; demand for investment
G: Government purchases of goods and services
Y_d: Aggregate (total) demand of goods and services in an economy (C + I + G)
R: Interest rate
M_d: Demand for money (demand for real balances)
M: Supply of money
P: Price level
b: Marginal propensity to consume; 0 < b < 1
h: Responsiveness of investment spending to R

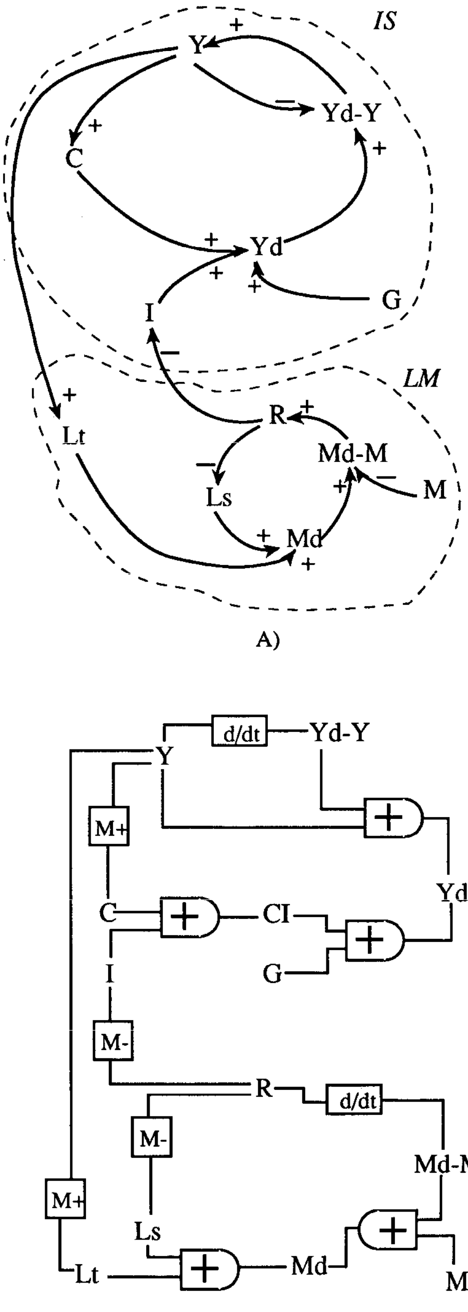

Figure 1. A) Causal diagram of the IS-LM model; B) Corresponding QSIM diagram.

Variables derived from the previous ones:

Lt: kY; demand for real balances for expenditure (transactions demand)

Ls: -m R; demand for real balances for holding money (speculative demand)

Parameters *k* (k > 0) and *m* reflect the sensitivity of the demand for real balances to the level of income and the interest rate, respectively.

B. Goods Market (IS curve).

Given a series of parameters, the IS model represents the goods sector within the Keynesian theory for the determination of output (or income) level of an economy.

This sector is formulated according to the following economic hypotheses and corresponding equations:

• (1) $C = c_0 + b\,Y$. C is the consumption function, which relates the demand for consumption goods to the level of income, Y, within the economy. For reasons of simplicity, we shall ignore taxes and transfers. The demand for consumption increases along with the level of income.

• (2) $I = r_0 - hR$. I is the planned investment spending. The higher the interest rate, the lower the desired or planned rate of investment. An increase in interest rate reduces spending on investment.

• (3) $G = g_0$. Government purchases of goods and services are assumed to be constant.

• (4) $Y = Y_d$ $(Y_d = C + I + G)$. This equation establishes that output is at its equilibrium level when the quantity of output produced, Y, is equal to the quantity demanded Y_d. In general, the quantity of goods demanded, or aggregate demand, depends on the level of income within the economy and on interest rates. There is one single level of equilibrium output at which the aggregate (total) demand for goods and services is equal to the level of output. An increase in interest rate reduces aggregate demand at a given level of income since an increase in interest rate reduces spending on investment. In this sector the rate of interest, R, is considered exogenous.

The goods market equilibrium schedule (IS schedule) shows combinations of interest rates and levels of output where planned spending equals income (goods market in equilibrium).

C. Money Market (LM curve).

The equations and hypotheses underlying this sector are:

• (5) $M_d = P\,(kY - mR)$. Real money balances are the quantity of nominal money divided by the price level, and the real demand for money is called the demand for real balances (M_d). The demand for real balances depends on the level of real income and the interest rate. It increases with the level of real income (kY) and decreases with the interest rate (-mR). We assume the price level is constant.

• (6) $M = M_0$. The nominal quantity of money, M, is controlled by the Federal Reserve System, and we take it as given at the level M_0.

• (7) $M_d = M_0$. The money market equilibrium schedule, or LM curve, shows combinations of interest rate and income levels such that the demand for real balances exactly matches the available supply. The equilibrium implies that an increase in the level of income (Y) is accompanied by an increase in the interest rate.

D. IS-LM Goods and Money Market.

The adjustment processes can be expressed as a set of two linear first order differential equations [10, Ch. 9] (eliminating constants):

$$dY/dt = Y_d - Y = \quad (b-1)\,Y \quad - h\,R$$
$$dR/dt = M_d - M = \quad k\,Y \quad - m\,R$$

The interest rate and level of output are jointly determined by simultaneous equilibrium of the goods and money markets.

Figure 2 shows an equilibrium in (r0, y0) through the intersection of IS and LM curves. Shifts in each of the curves will affect the equilibrium of the model and moves toward it. This model is used by economists to examine how the monetary and fiscal policy affect the economy.

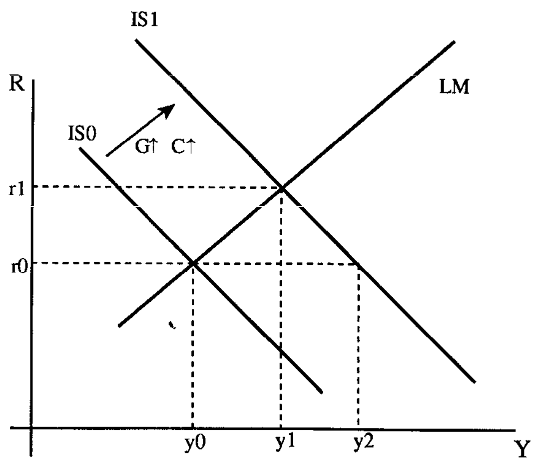

Figure 2. Initial equilibrium at (r0, y0). After shifting the G (or C) parameter, the IS0 curve moves to IS1 giving a new equilibrium point at (r1, y1). Taking only the IS sector into account, the new equilibrium would be at (r0, y2).

IV. Simulation and Loop Analysis

A. Results of Qualitative Simulation.

When proceeding to QS we should firstly have translated the relationships stated in section III into their qualitative form. Thus, the causal diagram will show all the relations as direct (+) or inverse (-). The QSIM diagram gives some of these relations in form of M^+ and M^- relations, and the remainder by means of 'addition' and 'derivative' constraints. Afterwards we will define the qualitative space for each variable.

Table I describes both structural equations and quantity-spaces for QS. For instance, in relation YdY = Y_d - Y (and dY/dt = YdY) there is a landmark at which dY/dt is 0, therefore making it 0 YdY, and meaning that it will be associated with some value belonging to Y, y0, with a value of Y_d, yd0. Obviously, the range of some variables, such as C, I, etc., must be over 0.

Each of the IS and LM sectors were, separately, qualitatively simulated, giving results as plotted in Figures 3 and 4.

Figure 3 shows how consumption (C), aggregate demand (Y_d) and output (Y) decrease after an initial state of disequilibrium (Y_d < Y). This is accomplished by setting Y and Y_d, respectively, at the intervals (y0 inf) and (yd0 inf), and setting the value of the difference Y_d - Y (ydy) at the interval (minf 0). Y_d decreases to yd0, and stays there. The same occurs with Y (output), which decreases to y0 and remains steady. The tree of behaviors indicates three possible behaviors which end in a state of equilibrium. On examining each behavior we find an equilibrium above y0 and another below y0. With the given initializations only the equilibrium in y0 is correct.

Figure 4 shows the result of "comparative statics" after shifting parameter M from a 'normal' state. An increase in money supply shifts the LM schedule

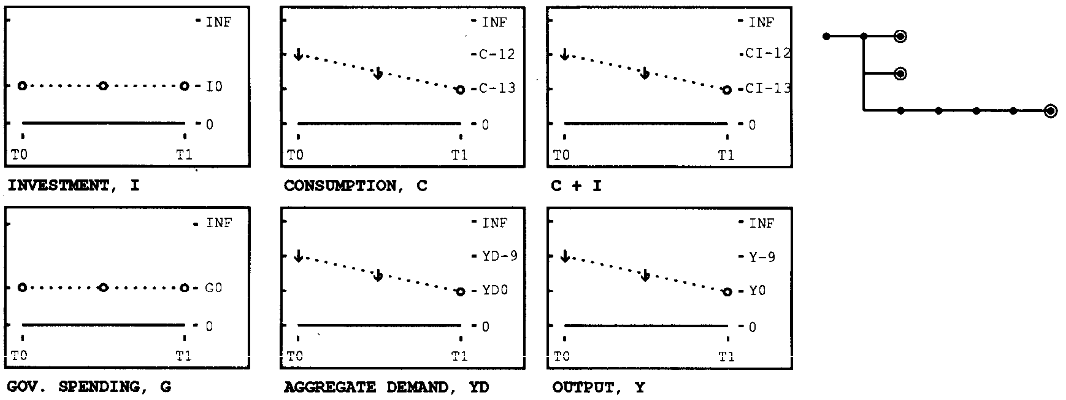

Figure 3. In these plots consumption, aggregate demand and output decrease after an initial state of disequilibrium Yd < Y.

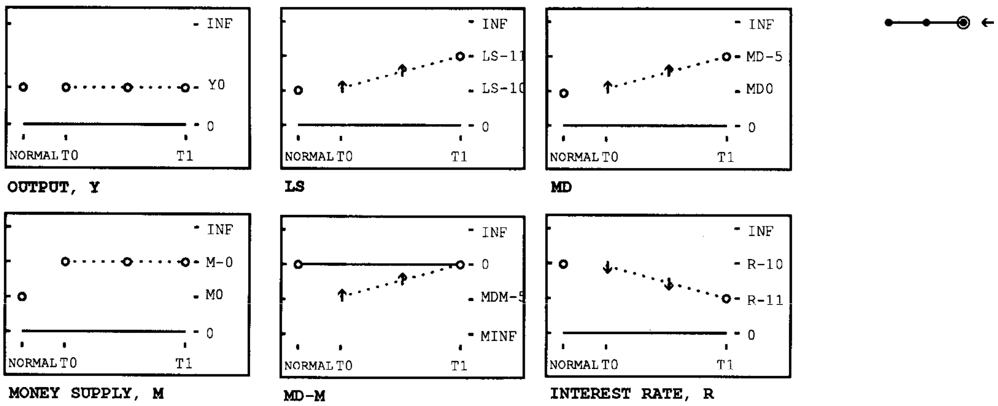

Figure 4. M is shifted from its 'normal' -M0- value to another value at (M0, INF).

downwards, and decreases the interest rate to maintain equilibrium. M is shifted in `T0` from `m0` to any value at interval `(m0 inf)` and is set to that value for the remainder of the simulation. The remaining variables move accordingly, and at time point `T1` they reach a new value of equilibrium The tree of behaviors shows one single way of reaching equilibrium.

VARIABLES & QUALITATIVE SPACES

Y	`(0 y0 inf)`	R	`(0 r0 inf)`
Y_d-Y	`(minf 0 inf)`	M_d-M	`(minf 0 inf)`
Y_d	`(0 yd0 inf)`	M_d	`(0 md0 inf)`
G	`(0 g0 inf)`	M	`(0 m0 inf)`
C	`(0 c0 inf)`	Lt	`(0 inf)`
I	`(0 i0 inf)`	Ls	`(0 inf)`
C+I	`(0 inf)`		

RELATION QS-CONSTRAINT

dY/dt = YdY	`((d/dt y ydy))`
YdY = Y_d - Y	`((add ydy y yd) (0 y0 yd0))`
Y_d = G + CI	`((add g ci yd))`
CI = C + I	`((add c i ci))`
C = k Y	`((m+ y c) (0 0) (y0 c0) (inf inf))`
I = -hR	`((m- r i) (0 inf) (r0 i0) (inf 0))`
dR/dt =MdM	`((d/dt r mdm))`
MdM = Md-M	`((mdm m md) (0 m0 md0))`
M_d = Lt + Ls	`((add lt ls) (0 m0 md0))`
Lt = kY	`((m+ y lt) (0 0) (inf inf))`
Ls = -mR	`((m- r i) (0 inf) (r0 i0) (inf 0))`

Table I. Variables and constraints for the model.

However, the results of the QS algorithms which are applied in the same way to the complete IS-LM give an intractable number of behaviors most of the time, although some cases could be interpreted as approaching to equilibrium.

The problem is that our system has the following Jacobian matrix:

$$\begin{bmatrix} - & - \\ + & - \end{bmatrix}$$

which, dynamically, has either a stable node or stable focus, preventing the local propagation of constraints from uniquely identify a path.

B. Loop Analysis.

To determine the complete qualitative properties of the second order system by means of LA, apply the formulas stated in section II, so as to obtain the global characterization of the system. Therefore, we must construct the loop diagram by identifying those variables whose rates of change could be affected. As we mentioned before, the loop diagram is slightly different from the causal diagram, and can be constructed once the variables that have variations in their rate of change have been identified. Since we have already constructed the differential equations, it is straightforward to describe the diagram as in Figure 5. The '+' sign means a positive or enhancing effect on the variable, and '-' represents a negative or decreasing effect on that variable. A self-effect link connects a variable to itself. This can be either self-enhancing (+), or self-damping (-).

Thus we proceed to calculations of stability and the prediction of changes.

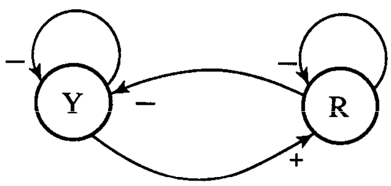

Figure 5. Loop diagram of the IS-LM model

• *Stability:* Feedback level (k=)

0 $F_0 = -1$ (by definition)

1 Addition of all loops of length 1

$F_1 = -a_{YY} - a_{RR} < 0$

2 Addition of all loops of length 2 and subtraction of all combinations of pairwise products of disjunct loops of length 1

$F_2 = -a_{YR}a_{RY} - ((-a_{YY})(-a_{RR})) < 0$

The system is *stable* since $F_i < 0$, for all i.

• *Qualitative Predictions*:

By applying the corresponding formula to the loop diagram we obtain in Table II the variations in variables Y and R which are produced by corresponding parameter variations.

Parameter change	Variation in rate of change of	Effect of parameter change in equilibrium of Y	R
↑ ɗ G	+ Y	+	+
↑M	− R	+	−

Table II

Once this table has been obtained it is not difficult to assign symbolic values to the new equilibrium point.

C. Comments.

Thus, in the IS-LM example, we observe that:

• the simplest modeling method is the causal diagram (and corresponding differential equation)

• the loop signed digraph, invaluable for LA, lacks the expressive power to follow the shift in all intermediate variables

• QS allows the symbolic manipulation of values, but generates multiple behaviors, meaning that it cannot be used systematically

• LA has already been applied since the model is suitable to that formulation.

From the systematic application point of view, it is correct to consider the combination of techniques, with aim to obtaining qualitative models in the area of social sciences, where some qualitative physics theories could not apply. Structural Modeling techniques [8] can help to define models, and also to establish a series of pattern models to which we can adjust newly designed models.

V. CONCLUSION

Conventional modeling methodologies exist which are adequate for formulating models for qualitative analysis. Due to the specific characteristics of QS, it can be easily applied to socieconomic areas, where good representation techniques already exist, meaning that ontological questions about qualitative physics can be avoided in these areas.

Classical qualitative analysis techniques can be readily integrated with new ones. Whilst techniques coming from AI have a greater capability for symbol manipulation, they lack of methods for grasping global behavior.

The presented model was delineated as a causal and signed digraph, in such a way as to be able to trace causality. Transformation to a QS program is straightforward. LA provides the global results of the model. Loop analysis is a valuable tool for QS provided that, in the modeling phase, we accommodate the model to a methodology for structuring models.

NOTE: qualitative simulation has been carried out with the Q system, which was kindly provided by Prof. Kuipers.

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