EQUALITY CONSTRAINT SOLVING

A dozen examples

The file agf.css contains the signature {a/0,f/2,g/1} which is used along the following examples. EC# means "Equality Constraint number #" which is inside the file EC#.cfs

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EC1:
∃W1(∃W2(∀Y1(∀Y2(f(f(W1,a),f(W2,X2)) ≠ f(f(Y1,a),f(Y2,Y2))   ∧
                     f(g(Y2),X1) ≠ f(X2,f(Y1,Y1))
                     ))))
Answers:
(X2 = _W1 ∧ _W1 ≠ g(*V1)) ∨
(X1 = _W1 ∧ _W1 ≠ f(*V1,*V1))

EC2:
∀Y1(∃W1(∀Y2(∃W2(f(X1,g(X2)) = f(W1,X1)   ∧
                 W1 ≠ f(Y1,Y2) ∧
                 W1 ≠ a ∧
                 f(X2,a) ≠ f(g(W2),X1)
                 ))))
Answers:
(X1 = g(_W1) ∧ X2 = _W1)

EC3:
∀Y1(∃W1(f(X1,g(X2)) = f(W1,X1) ∧ W1 ≠ g(Y1) ∧ W1 ≠ a))
Answers:
false

EC4:
∀Y1(∃W1(∀Y2(f(X1,g(X2)) = f(f(W1,X2),X3)   ∧
            W1 ≠ f(Y1,Y2) ∧ W1 ≠ a ∧
            ∃W2(∀Y3(f(X2,a) ≠ f(g(Y3),W1)))
            )))
Answers:
  (X3 = g(_W2) ∧ X1 = f(_W1,_W2) ∧ X2 = _W2 ∧ _W1 ≠ a ∧ _W1 ≠ f(*V1,*V2))

EC5:
∀Y1(∃W1(∀Y2(f(X1,g(X2)) = f(f(W1,X2),X3)   ∧
            W1 ≠ f(Y1,Y2) ∧ W1 ≠ a ∧
            ∃W2(∀Y3(f(X2,a) ≠ f(g(Y3),W1))) ∧
            ∀Y3(∀Y4(f(X1,X2) ≠ f(g(X3),f(Y3,Y4))   ∧
                    f(X3,Y1) = f(g(a),Y1)
                    ))
            )))
Answers:
  (X3 = g(a) ∧ X1 = f(_W1,a) ∧ X2 = a ∧ _W1 ≠ f(*V1,*V2) ∧ _W1 ≠ a)

EC6:
```
∀Y1(∃W1(∀Y2(f(X1,g(X2)) = f(f(W1,X2),X3)  ∧
            W1 ≠ f(Y1,Y2) ∧
            W1 ≠ a ∧
            ∃W2(∀Y3( f(X2,a) ≠ f(g(Y3),W1))) ∧
            ∀Y3(∀Y4(f(X1,X2) ≠ f(g(X3),f(Y3,Y4))  ∧
                     (X3 ≠ f(Y3,Y4) ∨  (Y3 = a ∧ Y4=a))
                      ))
          )))
```
Answers:
```
false
```

EC7:
```
(∀V1 (∀V2( X1 = f(X2,g(X3)) ∧ X2 ≠ f(V1,V2) ∧ X1 ≠ g(V2)))) ↔ X1=X2
```
Answers:
```
( X1 = _W2 ∧ X2 = _W1 ∧ X3 = _W3 ∧ _W1 ≠ _W2 ∧ _W2 ≠ f(*V1,g(*V2)) ) ∨
( X1 = f(_W2,g(_W4)) ∧ X2 = _W1 ∧ X3 = _W3 ∧ _W1 ≠ f(_W2,g(_W4)) ∧ _W1 ≠ _W2 )
∨
( X1 = f(_W1,g(_W3)) ∧ X2 = _W1 ∧ X3 = _W2 ∧ _W2 ≠ _W3 ) ∨
( X1 = f(f(_W2,_W3),g(_W1)) ∧ X2 = f(_W2,_W3) ∧ X3 = _W1 )
```

EC8:
```
∃W1( ∃W2( ∀Y1( (f(X1,X2) = f(f(Y1,X3),g(W2)) ∧ g(X2) = g(f(Y1,W1))) )))
```
Answers:
```
false
```

EC9:
```
∀V1( X1 ≠ f(X2,g(X3)) ∧
     X1 ≠ a ∧
     X1 ≠ X2  ∧
     (X1 ≠ g(V1) ↔ X3 ≠ f(X2,V1))) ∧
∃W1(∃W2(f(X1,X2) = f(g(W1),f(W1,W2))))
```
Answers:
```
( X1 = g(_W1) ∧ X2 = f(_W1,_W2) ∧ X3 = f(f(_W1,_W2),_W1) )
```

EC10:
```
∀V1( X1 ≠ f(X2,g(X3)) ∧
     X1 ≠ a ∧
     X1 ≠ X2  ∧
     (X1 ≠ g(V1) ↔ X3 ≠ f(X2,V1)) ∧
∃W1(∃W2(f(X1,X2) = f(g(W1),f(W1,W2)))) ∧
∀Y(∃W(X1 ≠ g(Y) ∨ (Y = f(W,W) ∧ W=a )))
```
Answers:
```
( X1 = g(f(a,a)) ∧ X2 = f(f(a,a),_W1) ∧ X3 = f(f(f(a,a),_W1),f(a,a)) )
```

EC11:
∃W(X2 = g(W) ∧ ∀Y(X1 ≠ f(Y,Y) ∨ Y = a) ∧ X1=f(W,X2) ∧
∀Y(X2 ≠ g(Y) ∨ ∃W2(Y=g(W2))) ∧
∀Y(X1 ≠ f(g(a),g(g(Y))))))
Answers:
(X1 = f(g(_W1),g(g(_W1))) ∧ X2 = g(g(_W1)) ∧ _W1 ≠ a)

EC12:
∀Y1(∃W1(∃W2(∀Y2(∃W3(f(g(X1),W1) = f(g(f(X2,X3)),W2)   ∧
                     (X3   ≠   g(Y1) ∨ Y1=a) ∧
                     (X1 =f(Y1,Y2) ∨ Y1 = Y2)
                    )))))
Answers:
false