Análisis Matemático

Resolver las siguientes ecuaciones:

1º $xdy + (xy + 2y - 2e^{-x})dx = 0$ con la condición $y(1)=0$.

2º $e^{y}\,\mathrm{sen}2x\,dx + (\cos x\, e^{2y} - y\cos x)dy = 0$.

3º $-y^{2}dx + (x^{2} + xy)dy = 0$. Buscando un factor integrante $\mu = \mu(x^{2}.y)$.

4º Resolver el sistema de ecuaciones diferenciales:

$$\left.\begin{aligned} \frac{dx}{dt} &= 2x + y + t - 2 \\ \frac{dy}{dt} &= 3x + 4y + e^{t} \end{aligned}\right\}$$

5º Aplicando Transformada de Laplace resolver:

$y'' + 4y' + 6y = 1 + e^{-t}$, $y(0)=y'(0)=0$.

6º Desarrollar en serie de Fourier:

$$f(x) = \begin{cases} \mathrm{sen}x & 0 \le x \le \pi \\ -\mathrm{sen}x & \pi < x < 2\pi \end{cases}$$

Solución

1º

$$y' + \left(\frac{x+2}{x}\right)y - \frac{2e^{-x}}{x} = 0$$ lineal, también se puede resolver con factor integrante $\mu = \mu$

$$y = u.v \Rightarrow u'v + uv' + \left(\frac{x+2}{x}\right)uv - \frac{2e^{-x}}{x} = 0; v\left(u' + \left(\frac{x+2}{x}\right)u\right) + uv' - \frac{2e^{-x}}{x} = 0; u' + \left(\frac{x+2}{x}\right)u$$

$$\ln u = -x - 2\ln x \Rightarrow u = \frac{e^{-x}}{x^2};$$

$$\frac{e^{-x}}{x^2}v' = \frac{2e^{-x}}{x} \Rightarrow v = x^2 + C.$$

$$y = \frac{e^{-x}}{x^2}(x^2 + C); y(1) = 0 = e^{-1}(1 + C) \Rightarrow C = -1.$$

$$y = \frac{e^{-x}}{x^2}(x^2 - 1).$$

2º

$$e^y \operatorname{sen}2x dx + \cos x (e^{2y} - y)dy = 0 \Rightarrow \frac{\operatorname{sen}2x}{\cos x}dx + \frac{e^{2y} - y}{e^y}dy = 0.$$

$$\int \frac{2\operatorname{sen}x\cos x}{\cos x}dx + \int \left(e^y - ye^{-y}\right)dy = C.$$

$$-2\cos x + e^y + e^{-y}(y + 1) = C.$$

3º

$$x^2y = t \Rightarrow \mu = \mu(t) \Rightarrow \begin{cases} \mu_x = 2xy\mu' \\ \mu_y = x^2\mu' \end{cases}$$

$$x^2\mu'P + \mu P_y = 2xy\mu'Q + \mu Q_x \Rightarrow \frac{\mu'}{\mu} = \frac{Q_x - P_y}{x^2P - 2xyQ} = \frac{2x + 3y}{-x^2y(2x + 3y)} = -\frac{1}{x^2y} = -\frac{1}{t}$$

$$\ln\mu = -\ln t \Rightarrow \mu = \frac{1}{t} = \frac{1}{x^2y}.$$

$$-\frac{y}{x^2}dx + \frac{(x + y)}{xy}dy = 0 \text{ exacta.}$$

$$F(x,y) = \frac{y}{x} + a(y) \Rightarrow \frac{\partial F}{\partial y} = \frac{1}{x} + a'(y) = \frac{(x + y)}{xy} = \frac{1}{y} + \frac{1}{x} \Rightarrow a'(y) = \frac{1}{y} \Rightarrow a(y) = \ln y.$$

$$\frac{y}{x} + \ln y = \ln C \Rightarrow \frac{y}{C} = e^{-\frac{y}{x}} \Rightarrow y = Ce^{-\frac{y}{x}}.$$

4º

$$x'' = 2x' + y' + 1 = 2x' + (3x + 4y + e^t) + 1;$$
$$y = x' - 2x - t + 2 \text{ sustituyendo}$$
$$x'' = 2x' + 3x + 4(x' - 2x - t + 2) + e^t + 1 \Rightarrow x'' - 6x' + 5x = e^t - 4t + 9$$
$$r^2 - 6r + 5 = 0; r = 1,5 \Rightarrow x_H = Ae^t + Be^{5t}.$$
$$x_{p1} = C\,te^t, \text{obligando a cumplir la ecuación} \Rightarrow C = -\frac{1}{4}.$$
$$x_{p2} = D + E\,t, \text{obligando a cumplir la ecuación} \Rightarrow D = \frac{21}{25}, E = -\frac{4}{5}.$$
$$x = Ae^t + Be^{5t} - \frac{1}{4}te^t + \frac{21}{25} - \frac{4}{5}t.$$
$$y = x' - 2x - t + 2 = Ae^t + 5Be^{5t} - \frac{1}{4}(1+t)e^t - \frac{4}{5} - 2\left(Ae^t + Be^{5t} - \frac{1}{4}te^t + \frac{21}{25} - \frac{4}{5}t\right) - t + 2 =$$
$$= -Ae^t + 3Be^{5t} + \frac{1}{4}(t-1)e^t + \frac{3}{5}t - \frac{12}{25}.$$

5º

$$L[y''] + 4L[y'] + 6L[y] = L[1 + e^{-t}] = \frac{1}{s} + \frac{1}{s+1};$$
$$(s^2 + 4s + 6)L[y] = \frac{1}{s} + \frac{1}{s+1} \Rightarrow L[y] = \frac{1}{s(s^2+4s+6)} + \frac{1}{(s+1)(s^2+4s+6)}$$
$$y = L^{-1}\left[\frac{2s+1}{s(s+1)(s^2+4s+6)}\right] = L^{-1}\left[\frac{A}{s} + \frac{B}{(s+1)} + \frac{Cs+D}{(s^2+4s+6)}\right]$$
$$y = \frac{1}{6} + \frac{1}{3}e^{-t} - e^{-2t}\left(\frac{\cos\sqrt{2}\,t}{2} + \frac{\sqrt{2}\,\mathrm{sen}\sqrt{2}\,t}{3}\right).$$

6º

Es una función “par” donde $2l = 2\pi$.

$$a_0 = \frac{2}{\pi}\int_0^\pi f(x)dx = \frac{2}{\pi}\int_0^\pi \mathrm{sen}x\,dx = \frac{2}{\pi}(-\cos x)\Big|_0^\pi = \frac{2}{\pi}(1+1) = \frac{4}{\pi}.$$
$$a_k = \frac{2}{\pi}\int_0^\pi f(x)\cos kx\,dx = \frac{2}{\pi}\int_0^\pi \mathrm{sen}x\cos kx\,dx = \frac{2}{\pi}\int_0^\pi \frac{\mathrm{sen}(1+k)x + \mathrm{sen}(1-k)x}{2}dx =$$

$$= \frac{1}{\pi}\int_0^{\pi}\left(\text{sen}(k+1)x - \text{sen}(k-1)x\right)dx = \frac{1}{\pi}\left(\frac{-\cos(k+1)x}{k+1} + \frac{\cos(k-1)x}{k-1}\right)\Big|_0^{\pi} =$$

$$= \frac{1}{\pi}\left[\frac{1}{k+1}(1-\cos(k+1)\pi - \frac{1}{k-1}(1-\cos(k-1)\pi\right] \quad \forall k \neq 1$$

$$a_1 = \frac{2}{\pi}\int_0^{\pi}\text{sen}x\cos x\,dx = \frac{2}{\pi}\int_0^{\pi}\frac{\text{sen}2x}{2}\,dx = \frac{1}{\pi}\left(\frac{-\cos 2x}{2}\right)\Big|_0^{\pi} = 0.$$

$$a_2 = \frac{1}{\pi}\left[\frac{2}{2+1} - \frac{2}{2-1}\right]$$

$$a_3 = 0.$$

$$a_4 = \frac{1}{\pi}\left[\frac{2}{4+1} - \frac{2}{4-1}\right]$$

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$$a_{2k} = \frac{1}{\pi}\left[\frac{2}{2k+1} - \frac{2}{2k-1}\right] = \frac{2}{\pi}\left[\frac{2k-1-(2k+1)}{4k^2-1}\right] = -\frac{4}{\pi}\left[\frac{1}{4k^2-1}\right] \quad k = 1,2,3...$$

$$f(x) = \frac{2}{\pi} - \frac{4}{\pi}\sum_{n=1}^{\infty}\frac{1}{4n^2-1}\cos 2nx.$$

Siendo “x” punto de continuidad.

No tiene puntos de discontinuidad.