Análisis Matemático

1º.- Hallar:

$$L\left[e^{-3t}.t^5+e^t.\text{sen}^2 t\right].$$

2º.- Calcular:

$$\int_0^\infty \frac{e^{-3t}\text{sen}^2 2t}{t}dt.$$

3º.- Resolver el sistema:

$$\left.\begin{array}{l}\dfrac{dx}{dt}=-7x+y+5\\ \dfrac{dy}{dt}=-2x-5y-37t\end{array}\right\}\ x(0)=y(0)=0.$$

Solución

1º.- $L\left[e^{-3t}.t^5\right]=\frac{5!}{(s+3)^6}$

$$L\left[e^{t}sen^2t\right]=L\left[e^{t}\left(\frac{1-\cos 2t}{2}\right)\right]=\frac{1}{2}\left[\frac{1}{s-1}-\frac{s-1}{(s-1)^2+4}\right].$$

El resultado la suma de las dos.

2º.- $\int_0^{\infty}\frac{e^{-3t}sen^2 2t}{t}dt=\lim_{s\to 3}L\left[\frac{sen^2 2t}{t}\right].$

$$\lim_{t\to 0}\frac{sen^2 2t}{t}=\text{infinitésimos equivalentes}=\lim_{t\to 0}\frac{(2t)^2}{t}=0\Rightarrow$$

$$\Rightarrow L\left[\frac{sen^2 2t}{t}\right]=\int_s^{\infty}\frac{1}{2}\left(\frac{1}{u}-\frac{u}{u^2+16}\right)du\ \ (*)$$

$$\text{pues } L\left[sen^2 2t\right]=L\left[\frac{1-\cos 4t}{2}\right]=\frac{1}{2}\left(\frac{1}{s}-\frac{s}{s^2+16}\right).$$

$$(*)=\frac{1}{2}\left(\ln u-\frac{1}{2}\ln(u^2+16)\right)\Big|_s^{\infty}=\frac{1}{4}\ln\left(\frac{u^2}{u^2+16}\right)\Big|_s^{\infty}=0-\frac{1}{4}\ln\left(\frac{s^2}{s^2+16}\right)=\frac{1}{4}\ln\left(\frac{s^2+16}{s^2}\right).$$

$$\text{La integral pedida}=\frac{1}{4}\ln\left(\frac{9+16}{9}\right)=\frac{1}{2}\ln\left(\frac{5}{3}\right).$$

3º.-Aplicando la transformada de Laplace al sistema:

$$\left.\begin{array}{l} sL[x]-x(0)=-7L[x]+L[y]+\dfrac{5}{s} \\ sL[y]-y(0)=-2L[x]-5L[y]-\dfrac{37}{s^2} \end{array}\right\} \Rightarrow \left.\begin{array}{l} (s+7)L[x]-L[y]=\dfrac{5}{s} \\ 2L[x]+(s+5)L[y]=-\dfrac{37}{s^2} \end{array}\right\}$$

$$\begin{vmatrix} (s+7) & -1 \\ 2 & (s+5) \end{vmatrix} = (s+7)(s+5)+2 = s^2+12s+37.$$

Aplicando Cramer :

$$L[x]=\frac{\begin{vmatrix} \frac{5}{s} & -1 \\ -\frac{37}{s^2} & (s+5) \end{vmatrix}}{s^2+12s+37}=\frac{\frac{5(s+5)}{s}-\frac{37}{s^2}}{s^2+12s+37}=\frac{5s^2+25s-37}{s^2(s^2+12s+37)}=\frac{A}{s}+\frac{B}{s^2}+\frac{Cs+D}{s^2+12s+37}\Rightarrow$$

$$\Rightarrow As(s^2+12s+37)+B(s^2+12s+37)+s^2(Cs+D)=5s^2+25s-37,$$

$$\left.\begin{array}{l} s=0\Rightarrow 37B=-37\Rightarrow B=-1. \\ coef(s^3); A+C=0. \\ coef(s^2); 12A+B+D=5. \\ coef(s); 37A+12B=25\Rightarrow A=1 \end{array}\right\} C=-1, D=-6.$$

$$x=L^{-1}\left[\frac{1}{s}\right]+L^{-1}\left[\frac{-1}{s^2}\right]+L^{-1}\left[\frac{-s-6}{s^2+12s+37}\right]=1-t+L^{-1}\left[\frac{-(s+6)}{(s+6)^2+1}\right]=1-t-e^{-6t}\cos t.$$

$$L[y]=\frac{\begin{vmatrix} s+7 & \frac{5}{s} \\ 2 & -\frac{37}{s^2} \end{vmatrix}}{s^2+12s+37}=\frac{\frac{-37(s+7)}{s^2}-\frac{10}{s}}{s^2+12s+37}=\frac{-47s-37.7}{s^2(s^2+12s+37)}=\frac{A}{s}+\frac{B}{s^2}+\frac{Cs+D}{s^2+12s+37}\Rightarrow$$

$$\Rightarrow As(s^2+12s+37)+B(s^2+12s+37)+s^2(Cs+D)=-47s-37.7$$

$$\left.\begin{array}{l} s=0 \Rightarrow 37B=-37.7 \Rightarrow B=-7. \\ coef(s^3); A+C=0. \\ coef(s^2); 12A+B+D=0. \\ coef(s); 37A+12B=-47 \Rightarrow A=1 \end{array}\right\} C=-1, D=-5.$$

$$y=L^{-1}\left[\frac{1}{s}\right]+L^{-1}\left[\frac{-7}{s^2}\right]+L^{-1}\left[\frac{-s-5}{s^2+12s+37}\right]=1-7t+L^{-1}\left[\frac{-(s+6)+1}{(s+6)^2+1}\right]=1-7t+e^{-6t}(-\cos t+\operatorname{sen} t)$$