# Análisis Matemático

Resolver las siguientes ecuaciones diferenciales:

**1º.-** $\frac{dy}{dx} + y = y^2(\cos x - \operatorname{sen} x).$

**2º.-** $(x + y - 1)\,dx + (y - 3x - 5)\,dy = 0.$

**3º.-** $y\,(x^2 + y^2 + 1)\,dx + x\,(1 - x^2 - y^2)\,dy = 0. \qquad \mu = \mu\,(x.y)$

1º.- Es una ecuación de Bernoulli.

$$\text{Dividiendo entre } y^2 \Rightarrow \frac{y'}{y^2}+\frac{1}{y}=\cos x-\operatorname{sen}x \Rightarrow \begin{cases} \frac{1}{y}=z \\ -\frac{y'}{y^2}=z' \end{cases} \text{ nos queda la lineal:}$$

$$-z'+z=\cos x-\operatorname{sen}x,\ \text{haciendo } z=u.v \Rightarrow -(u'v+uv')+uv=\cos x-\operatorname{sen}x$$

$$v(-u'+u)-uv'=\cos x-\operatorname{sen}x. \quad (*)$$

$$-u'+u=0 \Rightarrow \frac{du}{u}=dx; \ln u=x \Rightarrow u=e^x.$$

$$\text{Sustituyendo en } (*) \quad -e^x v'=\cos x-\operatorname{sen}x \Rightarrow dv=(\operatorname{sen}x-\cos x)e^{-x}dx.$$

$$\int \operatorname{sen}x\, e^{-x}dx=-e^{-x}\operatorname{sen}x+\int \cos x\, e^{-x}dx \quad (**).$$

$$v=\int \operatorname{sen}x\, e^{-x}dx-\int \cos x\, e^{-x}dx=\text{sustituyendo } (**)=-e^{-x}\operatorname{sen}x+K.$$

$$\text{Deshacienco cambios: } z=\frac{1}{y}=e^x(-e^{-x}\operatorname{sen}x+K)=-\operatorname{sen}x+Ke^x.$$

$$2º.-\ y'=\frac{x+y-1}{3x-y+5} \qquad \begin{cases} x+y=1 \\ 3x-y=-5 \end{cases} \Rightarrow x=-1, \quad y=2.$$

$$\text{Cambio } \begin{cases} x=X-1 \\ y=Y+2 \end{cases} \qquad y'=Y' \Rightarrow Y'=\frac{X+Y}{3X-Y} \text{ homogénea}$$

$$Y=u.X \Rightarrow Y'=u'X+u=\frac{X+uX}{3X-uX}=\frac{1+u}{3-u}.$$

$$u'X=\frac{1+u}{3-u}-u=\frac{1+u-3u+u^2}{3-u} \Rightarrow \frac{3-u}{u^2-2u+1}du=\frac{dX}{X}$$

$$\ln X=\int \frac{3-u}{(u-1)^2}du.$$

$$\frac{3-u}{(u-1)^2}=\frac{A}{u-1}+\frac{B}{(u-1)^2}\Rightarrow\begin{cases}A=-1\\B=2\end{cases}$$

$$\ln X=-\ln(u-1)-\frac{2}{u-1}+\ln C\Rightarrow\frac{X(u-1)}{C}=e^{\frac{-2}{u-1}}\Rightarrow$$

$$\Rightarrow\frac{X\left(\frac{Y}{X}-1\right)}{C}=e^{\frac{-2}{\frac{Y}{X}-1}}\Rightarrow Y-X=C\,e^{\frac{-2X}{Y-X}}\Rightarrow$$

$$\Rightarrow(y-2)-(x+1)=Ce^{\frac{-2(x+1)}{(y-2)-(x+1)}}$$

$$\Rightarrow(y-2)-(x+1)=Ce^{\frac{-2(x+1)}{(y-2)-(x+1)}}\Rightarrow y-x-3=Ce^{\frac{-2(x+1)}{y-x-3}}$$

3º.- $P=y(x^2+y^2+1)\Rightarrow P_y=x^2+y^2+1+2y^2=x^2+3y^2+1.-$

$Q=x(1-x^2-y^2)\Rightarrow Q_x=1-x^2-y^2-2x^2=1-3x^2-y^2.$

$(\mu P)_y=(\mu Q)_x\Rightarrow\mu_yP+\mu P_y=\mu_xQ+\mu Q_x\ (*)$

$$t=x.y;\ \mu=\mu(t)\Rightarrow\begin{cases}\mu_x=\frac{d\mu}{dt}.\frac{\partial t}{\partial x}=y\frac{d\mu}{dt}\\ \mu_y=\frac{d\mu}{dt}.\frac{\partial t}{\partial y}=x\frac{d\mu}{dt}\end{cases}$$

Sustituyendo en (*) $x\mu'P+\mu P_y=y\mu'Q+\mu Q_x\Rightarrow\mu'(xP-yQ)=\mu(Q_x-P_y).$

$$\frac{\mu'}{\mu}=\frac{Q_x-P_y}{xP-yQ}=\frac{(1-3x^2-y^2)-(x^2+3y^2+1)}{xy(x^2+y^2+1)-yx(1-x^2-y^2)}=\frac{-4(x^2+y^2)}{2xy(x^2+y^2)}=\frac{-2}{xy}=\frac{-2}{t}\Rightarrow$$

$$\Rightarrow\frac{d\mu}{\mu}=\frac{-2}{t}dt\Rightarrow\ln\mu=-2\ln t\Rightarrow\mu=\frac{1}{t^2}=\frac{1}{x^2y^2}.$$

La ecuación ahora ya es exacta:

$$F(x,y)=\int\frac{1}{x^2y}(x^2+y^2+1)dx=\frac{x}{y}-\frac{y}{x}-\frac{1}{x\,y}+\alpha(y).$$

$$\frac{\partial F}{\partial y}=-\frac{x}{y^2}-\frac{1}{x}+\frac{1}{x\,y^2}+\alpha'(y)=\mu Q=\frac{1}{xy^2}(1-x^2-y^2)\Rightarrow\alpha'(y)=0\Rightarrow\alpha(y)=Cte.$$

Integral general: $\frac{x}{y}-\frac{y}{x}-\frac{1}{x\,y}+Cte=K$ o $\frac{x}{y}-\frac{y}{x}-\frac{1}{x\,y}+C=0.$