Análisis Matemático

Resolver las siguientes ecuaciones:

1º $\frac{dx}{dy} = \frac{y^3 - 3x}{y}$.

2º $\left(\frac{1}{1+y^2} + \cos x - 2xy\right)\frac{dy}{dx} = y(y + \operatorname{sen} x) \qquad y(0) = 1$.

3º $2x^2y'' - 3xy' - 3y = 1 + 2x + x^2$.

4º Hallar:

$L\left[\frac{\operatorname{sen}2t}{t}\right]; \ L\left[t\int_0^t \operatorname{sen}u\,du\right]$ (sin hacer la integral).

Enunciar las propiedades aplicadas.

5º Hallar:

$L^{-1}\left[\frac{2s^2+1}{s(s+1)(s-2)}\right]; L^{-1}\left[\frac{5s\,e^{-2s}}{(s+4)^3}\right]$.

6º Desarrollar en serie de Fourier:

$$f(x) = \begin{cases} 2x & 0 < x \leq 1 \\ -2x + 4 & 1 < x < 2 \end{cases}$$

Solución

1º

$$x' = y^2 - \frac{3x}{y} \text{ es lineal en "x".}$$

$$x = u.v \Rightarrow u'v + uv' = y^2 - \frac{3uv}{y}; u(v' + \frac{3v}{y}) + u'v = y^2$$

$$(v' + \frac{3v}{y}) = 0 \Rightarrow \frac{dv}{v} = \frac{-3dy}{y} \Rightarrow \ln v = -3\ln y \Rightarrow v = \frac{1}{y^3}.$$

$$u'v = y^2 \Rightarrow dv = y^5 dy \Rightarrow v = \frac{y^6}{6} + C.$$

$$x = \frac{1}{y^3}\left(\frac{y^6}{6} + C\right) = \frac{y^3}{6} + \frac{C}{y^3}.$$

También se puede resolver buscando un factor integrante:

$$ydx + (3x - y^3)dy = 0; P_y = 1, Q_x = 3 \Rightarrow \frac{Q_x - P_y}{P} = \frac{2}{y} \Rightarrow \mu = y^2.$$

$$y^3dx + (3xy^2 - y^5)dy = 0, \text{es exacta}.$$

$$F(x, y) = \int y^3dx = y^3x + \alpha(y) \Rightarrow \frac{\partial F}{\partial y} = 3y^2x + \alpha'(y) = Q \Rightarrow \alpha'(y) = -y^5$$

$$\alpha(y) = -\frac{y^6}{6} + cte \Rightarrow y^3x - \frac{y^6}{6} = C.$$

2º

$$y(y + \text{sen}x)dx + \left(2xy - \frac{1}{1 + y^2} - \cos x\right)dy = 0.$$

$$P = y(y + \text{sen}x) \Rightarrow P_y = 2y + \text{sen}x.$$

$$Q = 2xy - \frac{1}{1 + y^2} - \cos x \Rightarrow Q_x = 2y + \text{sen}x.$$

Es diferencial exacta.

$$F(x, y) = \int (y^2 + y\,\text{sen}x)dx = y^2x - y\cos x + \alpha(y).$$

$$F_y = 2yx - \cos x + \alpha'(y) = Q \Rightarrow \alpha'(y) = -\frac{1}{1 + y^2} \Rightarrow \alpha(y) = -\text{arctg}\,y + cte.$$

Integral general: $y^2x - y\cos x - \text{arctg}\,y = C.$

$$y(0) = 1 \Rightarrow -1 - \text{arctg}1 = -1 - \frac{\pi}{4} = C.$$

$$y^2x - y\cos x - \text{arctg}\,y = -1 - \frac{\pi}{4}.$$

3º Es una ecuación de Euler.

$$x = e^t \Rightarrow \frac{dy}{dx} = \frac{dy}{dt}.\frac{dt}{dx} = y_t'e^{-t}.$$

$y_x'' = (y_t'' - y_t')e^{-2t}$ sustituyendo $2y_t'' - 2y_t' - 3y_t' - 3y = 1 + 2e^t + e^{2t}$;

$$2y_t'' - 5y_t' - 3y = 1 + 2e^t + e^{2t}.$$

$$2r^2 - 5r - 3 = 0 \Rightarrow r = \begin{cases} -\frac{1}{2} \\ 3 \end{cases} \Rightarrow y_H = Ae^{-\frac{1}{2}t} + Be^{3t}.$$

Hay que hallar tres particulares :

$$y_{p1} = C \Rightarrow 0 - 0 - 3C = 1 \Rightarrow C = -\frac{1}{3}.$$

$$y_{p2} = De^t \Rightarrow 2De^t - 5De^t - 3De^t = 2e^t \Rightarrow D = -\frac{1}{3}; y_{p2} = -\frac{1}{3}e^t.$$

$$y_{p3} = Ee^{2t} \Rightarrow 2.4Ee^{2t} - 5.2Ee^{2t} - 3Ee^{2t} = e^{2t} \Rightarrow E = -\frac{1}{5}; y_{p3} = -\frac{1}{5}e^{2t}.$$

$$\mathbf{y =} Ae^{-\frac{1}{2}t} + Be^{3t} - \frac{1}{3} - \frac{1}{3}e^t - \frac{1}{5}e^{2t} = \frac{A}{\sqrt{x}} + Bx^3 - \frac{1}{3}(1 + x) - \frac{1}{5}x^2.$$

4º $L\left[\frac{sen2t}{t}\right]$;

$$\lim_{t\to 0}\left(\frac{sen2t}{t}\right) = 2 \Rightarrow L\left[\frac{sen2t}{t}\right] = \int_s^\infty \left(\frac{2}{z^2 + 4}\right)dz = arctg\left(\frac{z}{2}\right)\Big|_s^\infty = \frac{\pi}{2} - arctg\left(\frac{s}{2}\right).$$

$$L\left[t\int_0^t senudu\right] = (-1)^1 \frac{d}{ds}\left\{L\left[\int_0^t senudu\right]\right\} \ (*)$$

$$L\left[\int_0^t senudu\right] = \frac{L[sent]}{s} = \frac{1}{s(s^2 + 1)};$$

$$(*) = \frac{3s^2 + 1}{s^2(s^2 + 1)^2}.$$

5º $L^{-1}\left[\dfrac{2s^2+1}{s(s+1)(s-2)}\right]$;

$$\frac{2s^2+1}{s(s+1)(s-2)}=\frac{A}{s}+\frac{B}{s+1}+\frac{C}{s-2}\Rightarrow\begin{cases}A=-\dfrac{1}{2}\\ B=1\\ C=\dfrac{3}{2}\end{cases}$$

$$L^{-1}\left[\frac{2s^2+1}{s(s+1)(s-2)}\right]=-\frac{1}{2}+e^{-t}+\frac{3}{2}e^{2t}.$$

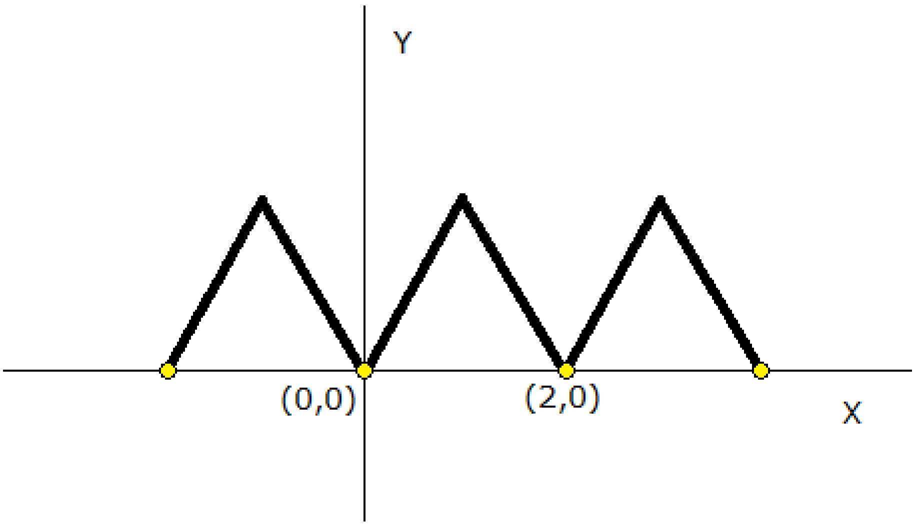
$$\frac{5s}{(s+4)^3}=\frac{5(s+4)-20}{(s+4)^3}=\frac{5}{(s+4)^2}-\frac{20}{(s+4)^3};$$

$$L^{-1}\left[\frac{5s}{(s+4)^3}\right]=5te^{-4t}-20\frac{t^2}{2}e^{-4t}=5te^{-4t}(1-2t).$$

Aplicando la traslación :

$$L^{-1}\left[\frac{5se^{-2s}}{(s+4)^3}\right]=\begin{cases}5(t-2)e^{-4(t-2)}\left(1-2(t-2)\right) & t>2\\ 0 & t<2\end{cases}$$

6º Es una función “par”.



$$a_0 = \frac{2}{1}\int_0^1 2x\,dx = 2x^2\Big|_0^1 = 2.$$

$$a_k = \frac{2}{1}\int_0^1 2x\cos k\pi x\,dx = 4\left[x\frac{\operatorname{sen} k\pi x}{k\pi} - \int \frac{\operatorname{sen} k\pi x}{k\pi}dx\right]\Bigg|_0^1 =$$

$$= 4\left[x\frac{\operatorname{sen} k\pi x}{k\pi} + \frac{\cos k\pi x}{(k\pi)^2}\right]\Bigg|_0^1 = 4\left[\frac{\cos k\pi}{(k\pi)^2} - \frac{1}{(k\pi)^2}\right] = \frac{4}{(k\pi)^2}(\cos k\pi - 1)$$

$$(\cos k\pi - 1) = \begin{cases} -2 & k = 1 \\ 0 & k = 2 \\ -2 & k = 3 \\ 0 & k = 4 \\ . & \\ . & \\ . & \end{cases}$$

x punto de continuidad:

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos n\pi x = 1 - \frac{8}{\pi^2}\sum_{n=1}^{\infty}\frac{\cos(2n-1)\pi x}{(2n-1)^2}.$$

$c = \{2k, k = 0, \pm 1, \pm 2....\}$ puntos de discontinuidad:

$$s(c) = \frac{0+0}{2} = 0 = 1 - \frac{8}{\pi^2}\sum_{n=1}^{\infty}\frac{\cos(2n-1)\pi c}{(2n-1)^2}.$$