Análisis Matemático

Resolver las siguientes ecuaciones diferenciales:

1º $\left(4x^2 + 3xy + y^2\right)dx + \left(4y^2 + 3xy + x^2\right)dy = 0$.

2º $xy' + (1 + x)y = e^{-x}\text{sen}2x$.

3º Resolver el sistema:

$$\left.\begin{aligned} \frac{dx}{dt} &= -7x + y + 5 \\ \frac{dy}{dt} &= -2x - 5y - 37t \end{aligned}\right\} \quad x(0) = y(0) = 0.$$

4º Hallar L[F(t)] siendo:

$$F(t) = \begin{cases} 0 & 0 \leq t < \dfrac{3\pi}{2} \\ \text{sen}t & t \geq \dfrac{3\pi}{2} \end{cases}$$

5º Calcular:

$$L^{-1}\left[\text{arctg}\left(\frac{3}{s}\right)\right];$$

1º Es una ecuación homogénea.

$$y' = -\frac{4x^2 + 3xy + y^2}{4y^2 + 3xy + x^2}; y = ux \Rightarrow y' = u'x + u = -\frac{4 + 3u + u^2}{4u^2 + 3u + 1} \Rightarrow$$

$$u'x = -\frac{4 + 3u + u^2}{4u^2 + 3u + 1} - u = \frac{-4u^3 - 4u^2 - 4u - 4}{4u^2 + 3u + 1} \Rightarrow \frac{4u^2 + 3u + 1}{u^3 + u^2 + u + 1}du = -\frac{4}{x}dx$$

$$u^3 + u^2 + u + 1 = (u + 1)(u^2 + 1)$$

$$\frac{4u^2 + 3u + 1}{u^3 + u^2 + u + 1} = \frac{A}{u + 1} + \frac{B + Cu}{u^2 + 1} \Rightarrow \begin{cases} A = 1 \\ B = 0 \\ C = 3 \end{cases}$$

$$\ln(u + 1) + \frac{3}{2}\ln(u^2 + 1) = -4\ln x + \ln K$$

$$(u + 1)^2(u^2 + 1)^3 = \frac{k^2}{x^8} \Rightarrow \left(\frac{y + x}{x}\right)^2 \left(\frac{y^2 + x^2}{x^2}\right)^3 = \frac{k^2}{x^8} \Rightarrow$$

$$(y + x)^2(y^2 + x^2)^3 = Cte.$$

2º Es una ecuación lineal.

$$y' + \left(\frac{1 + x}{x}\right)y = \frac{e^{-x}sen2x}{x}; y = uv \Rightarrow u'v + uv' + \left(\frac{1 + x}{x}\right)uv = \frac{e^{-x}sen2x}{x}$$

$$u\left(v' + \left(\frac{1 + x}{x}\right)v\right) + u'v = \frac{e^{-x}sen2x}{x} \text{ haciendo } \left(v' + \left(\frac{1 + x}{x}\right)v\right) = 0 \Rightarrow$$

$$\frac{dv}{dx} = -\left(\frac{1 + x}{x}\right)v = \left(-\frac{1}{x} - 1\right)v \Rightarrow \ln v = -\ln x - x \Rightarrow vx = e^{-x} \Rightarrow v = \frac{e^{-x}}{x}$$

$$u'\left(\frac{e^{-x}}{x}\right) = \frac{e^{-x}sen2x}{x} \Rightarrow du = sen2xdx \Rightarrow u = \frac{-\cos 2x}{2} + C$$

$$y = \left(\frac{-\cos 2x}{2} + C\right)\frac{e^{-x}}{x}$$

```
>>y= dsolve('t*Dy+(1+t)*y=exp(-t)*sin(2*t)')

   y =

   C5/(t*exp(t)) - cos(2*t)/(2*t*exp(t))

 >>y= simplify(y)

    y =

   (2*C5 - cos(2*t))/(2*t*exp(t))

 >> pretty(y
```

```
2 C5 - cos(2 t)
---------------
  2 t exp(t)
```

3º >> syms y;

$r^2 + 1 = 0 \Rightarrow r = \pm i \Rightarrow y_H = A\cos x + B\mathrm{sen}x$

$7\mathrm{sen}x.\mathrm{sen}2x = \frac{7}{2}(\cos x - \cos 3x)$

$y_{p_1} = x(C\cos x + D\mathrm{sen}x); y'_{p_1} = C\cos x + D\mathrm{sen}x + x(-C\mathrm{sen}x + D\cos x);$

$y''_{p_1} = 2(-C\mathrm{sen}x + D\cos x) + x(-C\cos x - D\mathrm{sen}x);$

$y''_{p_1} + y_{p_1} = 2(-C\mathrm{sen}x + D\cos x) + x(-C\cos x - D\mathrm{sen}x) + x(C\cos x + D\mathrm{sen}x) = \frac{7}{2}\cos x$

$$\Rightarrow \begin{cases} C = 0 \\ D = \frac{7}{4} \end{cases}$$

$y_{p_2} = (E\cos 3x + F\mathrm{sen}3x); y'_{p_2} = -3E\mathrm{sen}3x + 3F\cos 3x$

$y''_{p_2} = -9E\cos 3x - 9D\mathrm{sen}x$

$y''_{p_2} + y_{p_2} = -9E\cos 3x - 9F\mathrm{sen}3x + E\cos 3x + F\mathrm{sen}3x = -\frac{7}{2}\cos 3x$

$$\Rightarrow \begin{cases} E = \frac{7}{16} \\ F = 0 \end{cases}$$

$y = y_H + y_{p_1} + y_{p_2} = A\cos x + B\mathrm{sen}x + \frac{7}{4}x\mathrm{sen}x + \frac{7}{16}\cos 3x$

```
>> y=dsolve('D2y+y=7*sin(t)*sin(2*t)')

y =

sin(t)*((7*t)/4 - (7*sin(4*t))/16) - (49*cos(t)*(cos(t)/2 - 1/2)^2)/2 +
C5*cos(t) + C6*sin(t) - 14*cos(2*t)*(cos(t)/2 - 1/2)^2 -
(7*cos(3*t)*(cos(t)/2 - 1/2)^2)/2 - 14*(cos(t)/2 - 1/2)^2

>> y=simplify(y)
```

y =

(7*cos(t)^3)/4 - (7*cos(t))/4 + C5*cos(t) + C6*sin(t) + (7*t*sin(t))/4

>> pretty(y)

```
           3
 7 cos(t)      7 cos(t)                                     7 t sin(t)
 --------- - -------- + C5 cos(t) + C6 sin(t) + ----------
     4           4                                              4
```

4º

$$L[F(t)] = \int_{\frac{3\pi}{2}}^{\infty} e^{-st}\text{sent}dt \qquad (*)$$

$$\int e^{-st}\text{sent}dt = \frac{e^{-st}}{-s}\text{sent} + \frac{1}{s}\int e^{-st}\text{cost}dt =$$

$$= \frac{e^{-st}}{-s}\text{sent} + \frac{1}{s}\left(\frac{e^{-st}}{-s}\text{cost} + \frac{1}{s}\int e^{-st}(-\text{sent})dt\right) \Rightarrow$$

$$\left(1 + \frac{1}{s^2}\right)\int e^{-st}\text{sent}dt = -\frac{e^{-st}}{s^2}(s.\text{sent} + \text{cost}) \Rightarrow$$

$$\int e^{-st}\text{sent}dt = -\frac{e^{-st}}{s^2+1}(s.\text{sent} + \text{cost})$$

$$(*) = -\frac{e^{-st}}{s^2+1}(s.\text{sent} + \text{cost})\Big|_{\frac{3\pi}{2}}^{\infty} = \lim_{t\to\infty}\left[-\frac{e^{-st}}{s^2+1}(s.\text{sent} + \text{cost})\right] + \frac{e^{\frac{-3\pi s}{2}}}{s^2+1}(-s);$$

$$\text{si} \quad s > 0 \Rightarrow \lim_{t\to\infty}\left[-\frac{e^{-st}}{s^2+1}(s.\text{sent} + \text{cost})\right] = 0.\text{acotado} = 0.$$

$$L[F(t)] = -\frac{s.e^{\frac{-3\pi s}{2}}}{s^2+1}$$

5º

$$f(s) = \text{arctg}\left(\frac{3}{s}\right) \Rightarrow f'(s) = \frac{1}{1 + \frac{9}{s^2}}\left(\frac{-3}{s^2}\right) = \frac{-3}{s^2 + 9}$$

$$L^{-1}[f'(s)] = -\text{sen}3t = -t\,L^{-1}[f(s)] = -t\,L^{-1}\left[\text{arctg}\left(\frac{3}{s}\right)\right] \Rightarrow$$

$$L^{-1}\left[\text{arctg}\left(\frac{3}{s}\right)\right] = \frac{\text{sen}3t}{t}$$