Análisis Matemático

Resolver las siguientes ecuaciones:

1º $(2xy^2 + y)dx + (x + 2x^2y - x^4y^3)dy = 0.$ $\mu = \mu(x.y).$

2º $2x^3y' = y(y^2 + 3x^2).$

3º $x^2 y'' - 4xy' + 6y = -9x^2.$

4º Hallar la solución de la ecuación homogénea e indicar la "**forma**" de las soluciones particulares (sin resolver), en las siguientes ecuaciones:

a) $y^{IV} - 2y''' + 17y'' = 2x - 5 + 3e^x + (x^2 + x - 5)\text{sen}4x.$

b) $y''' - 2y'' + 5y' = 5xe^x + \cos^2 x - 3.$

5º Hallar: $\displaystyle\int_0^{\infty} e^{-t}\left(\frac{\text{sen}2t.\text{sen}3t}{t}\right)dt.$

6º Calcular: $L^{-1}\left[\dfrac{e^{-3s}}{(s^2 + 4s + 20)^2}\right]$ explicando las propiedades aplicadas.

Solución

1º

$$\left.\begin{array}{l} P = 2xy^2 + y \Rightarrow P_y = 4xy + 1. \\ Q = x + 2x^2y - x^4y^3 \Rightarrow Q_x = 1 + 4xy - 4x^3y^3. \end{array}\right\} t = xy \Rightarrow \frac{\mu_t'}{\mu} = \frac{Q_x - P_y}{xP - yQ} = \frac{-4}{xy} = \frac{-4}{t}$$

$$\Rightarrow \ln\mu = -4\ln t \Rightarrow \mu = \frac{1}{x^4y^4}.$$

$$\frac{2xy^2 + y}{x^4y^4}dx + \frac{x + 2x^2y - x^4y^3}{x^4y^4}dy = 0 \ \text{ exacta.}$$

$$F(x,y) = \int \frac{2xy^2 + y}{x^4y^4}dx = -\frac{1}{x^2y^2} - \frac{1}{3x^3y^3} + \alpha(y)$$

$$\frac{\partial F}{\partial y} = \frac{2}{x^2y^3} + \frac{1}{x^3y^4} + \alpha'(y) = Q = \frac{x + 2x^2y - x^4y^3}{x^4y^4} \Rightarrow \alpha'(y) = -\frac{1}{y} \Rightarrow \alpha(y) = -\ln y.$$

$$\text{Integral general}: -\frac{1}{x^2y^2} - \frac{1}{3x^3y^3} - \ln y = K = \ln C \Rightarrow yC = e^{-\frac{1}{x^2y^2}\left(1+\frac{1}{3xy}\right)}.$$

2º Es homogénea y de Bernoulli.

$$y' = \frac{y^3 + 3yx^2}{2x^3}; y = ux \Rightarrow u'x + u = \frac{u^3 + 3u}{2}$$

$$\frac{2du}{u^3 + u} = \frac{dx}{x} \Rightarrow 2\ln u - \ln(u^2 + 1) = \ln x + \ln k \Rightarrow \frac{u^2}{u^2 + 1} = xk \Rightarrow \frac{y^2}{x^2 + y^2} = xk.$$

Bernoulli.

$$2y' = \frac{y^3}{x^3} + \frac{3y}{x}; \qquad \frac{2y'}{y^3} = \frac{1}{x^3} + \frac{3}{xy^2}; \qquad z = \frac{1}{y^2} \Rightarrow -z' = \frac{1}{x^3} + \frac{3z}{x} \text{ lineal}$$

$$-(u'v + uv') = \frac{1}{x^3} + \frac{3uv}{x}; \quad u\underbrace{\left(-v' - \frac{3v}{x}\right)}_{0} - u'v = \frac{1}{x^3} \Rightarrow v = \frac{1}{x^3}$$

$$u = -x + C \Rightarrow z = \frac{1}{y^2} = u.v = \frac{-x + C}{x^3}; \quad \frac{x^2}{y^2} = -1 + \frac{C}{x}; \quad \frac{x^2 + y^2}{y^2} = \frac{C}{x};$$

3º

$$x = e^t \Rightarrow \begin{cases} y_x' = y_t' e^{-t} \\ y_x'' = (y_t'' - y_t')e^{-2t} \end{cases}$$

$$y_t'' - 5y_t' + 6y = -9e^{2t};\ r^2 - 5r + 6 = 0 \Rightarrow \begin{cases} r = 2 \\ r = 3 \end{cases}$$

$$y_H = Ae^{2t} + Be^{3t}.$$

$$y_p = tCe^{2t};\ y_p' = C(1+2t)e^{2t};\ y_p'' = C(4+4t)e^{2t}.$$

$$y_p'' - 5y_p' + 6y_p = -9e^{2t} \Rightarrow C = 9.$$

$$y = Ae^{2t} + Be^{3t} + 9te^{2t} = Ax^2 + Bx^3 + 9x^2 \ln x.$$

4º

a)

$$r^4 - 2r^3 + 17r^2 = 0 \Rightarrow \begin{cases} r = 0 \text{ doble} \\ r = 1 \pm 4i \end{cases}$$

$$y_H = A + Bx + e^x(C\cos 4x + D\,\mathrm{sen}\,4x).$$

$$y_{p1} = x^2(A_1 + A_2 x).$$

$$y_{p2} = B_1 e^x.$$

$$y_{p3} = (C_1 + C_2 x + C_3 x^2)\cos 4x + (D_1 + D_2 x + D_3 x^2)\,\mathrm{sen}\,4x.$$

b)

$$r^3 - 2r^2 + 5r = 0 \Rightarrow \begin{cases} r = 0 \\ r = 1 \pm 2i \end{cases}$$

$$y_H = A + e^x(B\cos 2x + C\,\mathrm{sen}\,2x).$$

$$y_{p1} = (A_1 + A_2 x)e^x.$$

$$\cos^2 x - 3 = \frac{1 + \cos 2x}{2} - 3 = -\frac{5}{2} + \frac{\cos 2x}{2}.$$

$$y_{p2} = xB_1.$$

$$y_{p3} = C_1\cos 2x + C_2\,\mathrm{sen}\,2x.$$

5º

$$= \lim_{s\to 1} L\left[\frac{sen2t.sen3t}{t}\right]; \lim_{t\to 0}\left(\frac{sen2t.sen3t}{t}\right) = 0 \Rightarrow L\left[\frac{sen2t.sen3t}{t}\right] =$$

$$= \int_s^{\infty} L[sen2u.sen3u]du. \; (\;*)$$

$$sen2u.sen3u = \frac{cosu - cos5u}{2}.$$

$$(\;*) = \frac{1}{2}\int_s^{\infty}\left(\frac{u}{u^2+1} - \frac{u}{u^2+25}\right)du = \frac{1}{4}ln\left(\frac{u^2+1}{u^2+25}\right)\Bigg|_s^{\infty} = \frac{1}{4}ln\left(\frac{s^2+25}{s^2+1}\right).$$

Resultado : $\frac{1}{4}ln13.$

6º

$$= \begin{cases} F(t-3) & t > 3 \\ 0 & 0 < t < 3 \end{cases} \quad (*) \quad \text{siendo } F(t) = L^{-1}\left[\frac{1}{(s^2+4s+20)^2}\right];$$

$$L^{-1}\left[\frac{1}{(s^2+4s+20)^2}\right] = L^{-1}\left[\frac{1}{((s+2)^2+16)^2}\right] = e^{-2t}L^{-1}\left[\frac{1}{(s^2+16)^2}\right].$$

$$f(s) = -\frac{1}{2}\left(\frac{1}{s^2+16}\right); f'(s) = \frac{s}{(s^2+16)^2} \Rightarrow L^{-1}\left[f'(s)\right] = -t\,L^{-1}\left[f(s)\right] = \frac{t\,sen4t}{8}.$$

$$L^{-1}\left[\frac{1}{(s^2+16)^2}\right] = \text{division entre "s"} = \int_0^t \frac{z\,sen4z}{8}dz =$$

$$\frac{1}{8}\left[\frac{-t\cos4t}{4} + \frac{sen4t}{16}\right] = \frac{1}{128}\left[sen4t - 4tcos4t\right].$$

$$(*) = \begin{cases} e^{-2(t-3)}\frac{1}{128}\left[sen4(t-3) - 4(t-3)cos4(t-3)\right] & t > 3 \\ 0 & 0 < t < 3 \end{cases}$$