CALCULO

1º Operar dejando el resultado en forma polar y binómica:

$$\frac{(\sqrt{3} - \sqrt{3}\ i)^6}{(\sqrt{15}\ i - \sqrt{5})^5(-2\sqrt{3} - 2i)^8}.$$

2º Sea:

$$f(x) = \begin{cases} \cos x - 1 & x \geq 0 \\ x^2 + 3x & x < 0 \end{cases}$$

Hallar $f^{(n}(x)$.

3º Si $z = e^{\frac{x}{y}}\operatorname{sen}\frac{x}{y} + e^{\frac{y}{x}}\cos\frac{y}{x}$, hallar : $x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y}$

4º Hallar: $\displaystyle\int \frac{1}{\cos^4 x.\operatorname{sen}^2 x}dx$

5º Invertir el orden de integración en:

$$\int_0^{1/2} dx \int_{1-\sqrt{1-x^2}}^{\sqrt{2x-x^2}} f(x,y)dy + \int_{1/2}^{\sqrt{3}/2} dx \int_{1-\sqrt{1-x^2}}^{\sqrt{1-x^2}} f(x,y)dy.$$

6º Hallar:

a) $\displaystyle\iiint_V z\,dx\,dy\,dz$. Donde V son los puntos situados por encima del plano $z=1$ y dentro de la esfera $x^2 + y^2 + z^2 = 2$.

b) Volumen de V.

Solución

1º

$$\sqrt{3} - \sqrt{3}\,i = \sqrt{6}_{\,315º}$$
$$\sqrt{15}\,i - \sqrt{5} = 2\sqrt{5}_{\,120º}$$
$$-2\sqrt{3} - 2i = 4_{\,210º}$$
$$\text{Operando}: \left(\frac{6^3}{2^5\,5^2\sqrt{5}\,4^8}\right)_{315º.6-5.120º-8.210º} = \left(\frac{3^3}{5^2\sqrt{5}\,2^{18}}\right)_{-30º} =$$
$$= \left(\frac{3^3}{5^2\sqrt{5}\,2^{19}}\right)(\sqrt{3} - i).$$

2º

$$f'(0^+) = \lim_{h\to 0}\left(\frac{\cos h - 1}{h}\right) = 0$$
$$f'(0^-) = \lim_{h\to 0}\left(\frac{(-h)^2 + 3(-h)}{-h}\right) = 3$$

No tiene derivada en x=0.

$$f'(x) = \begin{cases} -\operatorname{sen}x & x > 0 \\ 2x + 3 & x < 0 \end{cases}$$
$$f''(x) = \begin{cases} -\cos x & x > 0 \\ 2 & x < 0 \end{cases}$$
$$f'''(x) = \begin{cases} \operatorname{sen}x & x > 0 \\ 0 & x < 0 \end{cases}$$

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$$f^{(n}(x) = \begin{cases} \cos\left(x + n\frac{\pi}{2}\right) & x > 0 \\ 0 & x < 0 \end{cases} \qquad \forall n > 2$$

3º

$$\frac{\partial z}{\partial x} = e^{\frac{x}{y}}\frac{1}{y}\operatorname{sen}\left(\frac{x}{y}\right) + e^{\frac{x}{y}}\frac{1}{y}\cos\left(\frac{x}{y}\right) + e^{\frac{y}{x}}\left(\frac{-y}{x^2}\right)\cos\left(\frac{y}{x}\right) + e^{\frac{y}{x}}\left(\frac{y}{x^2}\right)\operatorname{sen}\left(\frac{y}{x}\right) =$$
$$= e^{\frac{x}{y}}\frac{1}{y}\left(\operatorname{sen}\left(\frac{x}{y}\right) + \cos\left(\frac{x}{y}\right)\right) + e^{\frac{y}{x}}\left(\frac{y}{x^2}\right)\left(-\cos\left(\frac{y}{x}\right) + \operatorname{sen}\left(\frac{y}{x}\right)\right).$$
$$\frac{\partial z}{\partial y} = e^{\frac{x}{y}}\left(\frac{-x}{y^2}\right)\operatorname{sen}\left(\frac{x}{y}\right) + e^{\frac{x}{y}}\left(\frac{-x}{y^2}\right)\cos\left(\frac{x}{y}\right) + e^{\frac{y}{x}}\left(\frac{1}{x}\right)\cos\left(\frac{y}{x}\right) - e^{\frac{y}{x}}\left(\frac{1}{x}\right)\operatorname{sen}\left(\frac{y}{x}\right) =$$

$$= e^{\frac{x}{y}}\left(\frac{-x}{y^2}\right)\left(\operatorname{sen}\left(\frac{x}{y}\right)+\cos\left(\frac{x}{y}\right)\right)+e^{\frac{y}{x}}\left(\frac{1}{x}\right)\left(\cos\left(\frac{y}{x}\right)-\operatorname{sen}\left(\frac{y}{x}\right)\right)$$

$$x\frac{\partial z}{\partial x}+y\frac{\partial z}{\partial y}=x\left[e^{\frac{x}{y}}\frac{1}{y}\left(\operatorname{sen}\left(\frac{x}{y}\right)+\cos\left(\frac{x}{y}\right)\right)+e^{\frac{y}{x}}\left(\frac{y}{x^2}\right)\left(-\cos\left(\frac{y}{x}\right)+\operatorname{sen}\left(\frac{y}{x}\right)\right)\right]+$$

$$y\left[e^{\frac{x}{y}}\left(\frac{-x}{y^2}\right)\left(\operatorname{sen}\left(\frac{x}{y}\right)+\cos\left(\frac{x}{y}\right)\right)+e^{\frac{y}{x}}\left(\frac{1}{x}\right)\left(\cos\left(\frac{y}{x}\right)-\operatorname{sen}\left(\frac{y}{x}\right)\right)\right]=0.$$

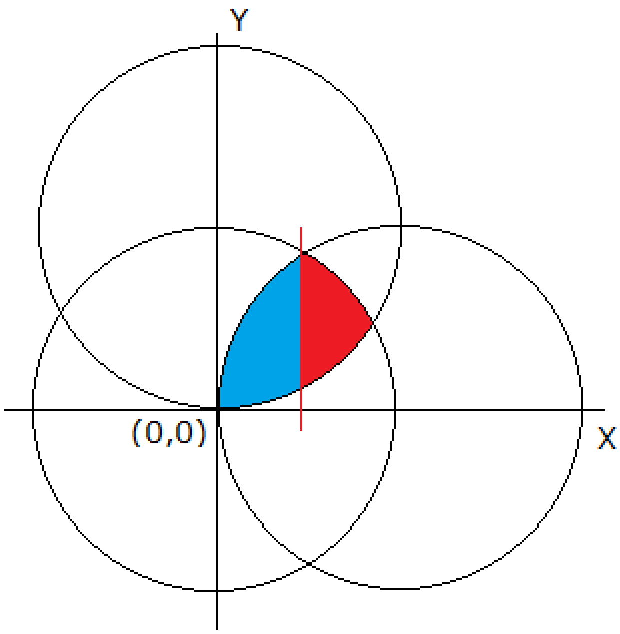
4º

$$\operatorname{tg}x = t \Rightarrow \frac{1}{\cos^2 x}dx = dt;\ 1+t^2=\frac{1}{\cos^2 x}$$

$$\int \frac{1}{\cos^4 x.\operatorname{sen}^2 x}dx = \int \frac{(1+t^2)^2}{t^2}dt = -\frac{1}{t}+2t+\frac{t^3}{3}+Cte =$$

$$= -\frac{1}{\operatorname{tg}x}+2\operatorname{tg}x+\frac{\operatorname{tg}^3 x}{3}+Cte.$$

5º



$$x^2 + (y-1)^2 = 1; (x-1)^2 + y^2 = 1; x^2 + y^2 = 1.$$

$$\int_0^{1/2} dy \int_{1-\sqrt{1-y^2}}^{\sqrt{2y-y^2}} f(x,y)\,dx + \int_{1/2}^{\sqrt{3}/2} dy \int_{1-\sqrt{1-y^2}}^{\sqrt{1-y^2}} f(x,y)\,dx.$$

6º a)

$$\iiint_V z\,dx\,dy\,dz = \iint_D dx\,dy \int_1^{\sqrt{2-x^2-y^2}} z\,dz = \frac{1}{2}\iint_D (2 - x^2 - y^2 - 1)\,dx\,dy$$

$D: x^2 + y^2 \leq 1$ pasando a coordenadas polares:

$$\frac{1}{2}\int_0^{2\pi} d\alpha \int_0^1 (1 - r^2) r\,dr = \frac{\pi}{4}$$

b)

$$\iiint_V dx\,dy\,dz = \iint_D dx\,dy \int_1^{\sqrt{2-x^2-y^2}} dz = \iint_D (\sqrt{2 - x^2 - y^2} - 1)\,dx\,dy$$

$D: x^2 + y^2 \leq 1$ pasando a coordenadas polares:

$$\int_0^{2\pi} d\alpha \int_0^1 (\sqrt{2 - r^2} - 1) r\,dr = 2\pi \frac{(4\sqrt{2} - 5)}{6} = \frac{\pi(4\sqrt{2} - 5)}{3}$$ unidades de volumen.