CÁLCULO

1º Sea:

$$f(x) = \begin{cases} \ln(2-x) & -\infty < x \le 1 \\ \sqrt{x-1} & x > 1 \end{cases}$$

Calcular $f^{(n}(x)$.

2º Hallar:

$$\lim_{x\to 0}\left[\operatorname{tg}\left(x+\frac{\pi}{4}\right)\right]^{\frac{1}{\operatorname{sen}x}}.$$

3º Hallar el dominio de la siguiente función y representarlo gráficamente

$$f(x,y) = \sqrt{\frac{4-x^2-y^2}{x-y}}.$$

4º Sea: $f(x,y) = x^2 \ln\left(\frac{x}{y}\right)$.

¿Se cumple $x^2 f_{xx} + 2xy f_{xy} + y^2 f_{yy} = 2f$?

5º $\displaystyle\int \frac{\sqrt{1+\ln x}}{x\ln x}\,dx.$

6º Calcular el volumen de la región del espacio limitada por el paraboloide $z+1=x^2+y^2$, el cilindro $x^2+y^2=4$ y el plano $z=-3$.

Solución

1º La función f(x) es continua en todo R.

$$f'(1^+) = \lim_{h\to 0}\frac{\sqrt{(1+h)-1}-0}{h} = \infty$$

$$f'(1^-) = \lim_{h\to 0}\frac{\ln(2-(1-h))-0}{-h} = \lim_{h\to 0}\frac{\ln(1+h)}{-h} = -1$$

No es derivable en x=1.

$$f'(x) = \begin{cases} -(2-x)^{-1} & x<1 \\ \frac{1}{2}(x-1)^{-\frac{1}{2}} & x>1 \end{cases} \Rightarrow f''(x) = \begin{cases} -(2-x)^{-2} & x<1 \\ -\frac{1}{2}.\frac{1}{2}(x-1)^{-\frac{3}{2}} & x>1 \end{cases}$$

$$f'''(x) = \begin{cases} -2(2-x)^{-3} & x<1 \\ \frac{1}{2}.\frac{1}{2}.\frac{3}{2}(x-1)^{-\frac{5}{2}} & x>1 \end{cases} \qquad ....$$

$$....f^{(n}(x) = \begin{cases} -(n-1)!(2-x)^{-n} & x<1 \\ (-1)^{n+1}\frac{1.3.5...(2n-3)}{2^n}(x-1)^{-\frac{(2n-1)}{2}} & x>1 \end{cases} \qquad n\geq 2$$

2º

$$\lim_{x\to 0}\left[tg\left(x+\frac{\Pi}{4}\right)\right]^{\frac{1}{senx}} = 1^{\infty} = e^{\lim_{x\to 0}\frac{1}{senx}\ln\left(tag(x+\frac{\Pi}{4})\right)} \quad (*)$$

$$\lim_{x\to 0}\frac{\ln\left(tag(x+\frac{\Pi}{4})\right)}{senx} = \frac{0}{0}\ (senx \sim x) = \lim_{x\to 0}\frac{\ln\left(tag(x+\frac{\Pi}{4})\right)}{x} = L\acute{}\,Hôp =$$

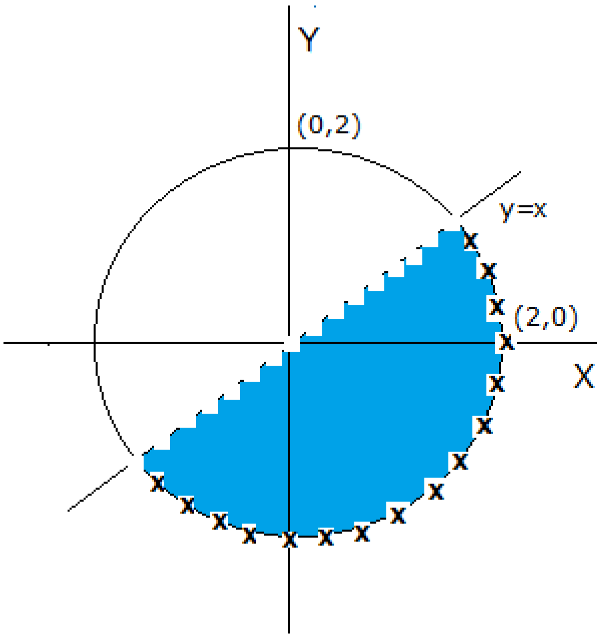
$$\lim_{x\to 0}\frac{\frac{1}{tag(x+\frac{\Pi}{4})}\frac{1}{\cos^2(x+\frac{\Pi}{4})}}{1} = 2.$$

$(*) = e^2.$

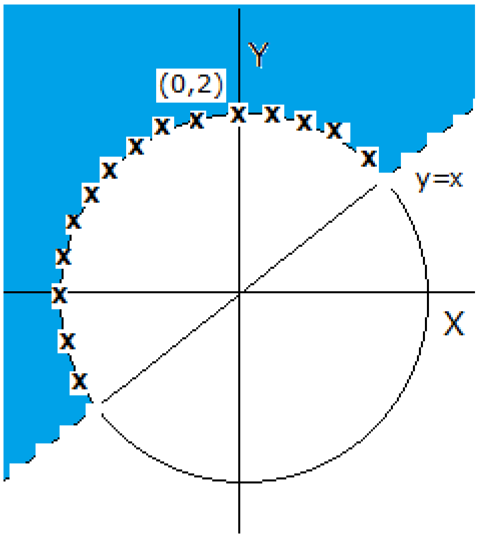
3º

$$D(f) = \left\{ (x, y) \in R^2 \ / \ \frac{4 - x^2 - y^2}{x - y} \geq 0 \ \text{ con } x \neq y \right\}$$

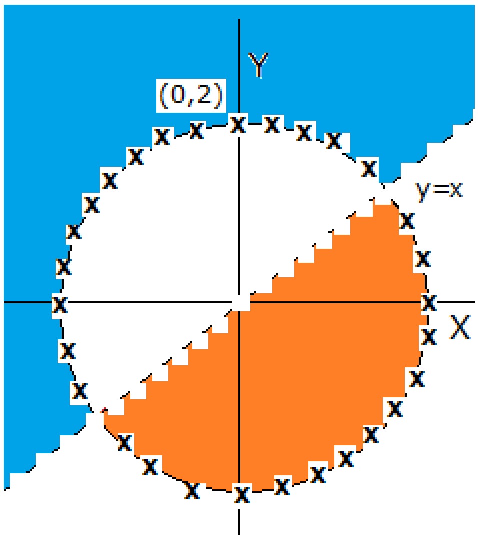
1º Caso: $4 - x^2 - y^2 \geq 0$ y $x - y > 0$



2º Caso: $4 - x^2 - y^2 \leq 0$ y $x - y < 0$



El resultado es la unión de los dos, el relleno naranja (1º caso) y el azul (2º caso):



4º

$$f_x = 2x\ln\left(\frac{x}{y}\right) + x^2.\frac{y}{x}.\frac{1}{y} = 2x\ln\left(\frac{x}{y}\right) + x.$$

$$f_{xx} = 2\ln\left(\frac{x}{y}\right) + 2x.\frac{y}{x}.\frac{1}{y} + 1 = 2\ln\left(\frac{x}{y}\right) + 3.$$

$$f_{xy} = 2x.\frac{y}{x}.\left(\frac{-x}{y^2}\right) = -2\frac{x}{y}.$$

$$f_y = x^2.\frac{y}{x}.\left(\frac{-x}{y^2}\right) = -\frac{x^2}{y}.$$

$$f_{yy} = \frac{x^2}{y^2}.$$

$$f_{xx} + 2xyf_{xy} + y^2f_{yy} = x^2\left(2\ln\left(\frac{x}{y}\right) + 3\right) + 2xy\left(-2\frac{x}{y}\right) + y^2\left(\frac{x^2}{y^2}\right) = 2x^2\ln\left(\frac{x}{y}\right) = 2f.$$

5º

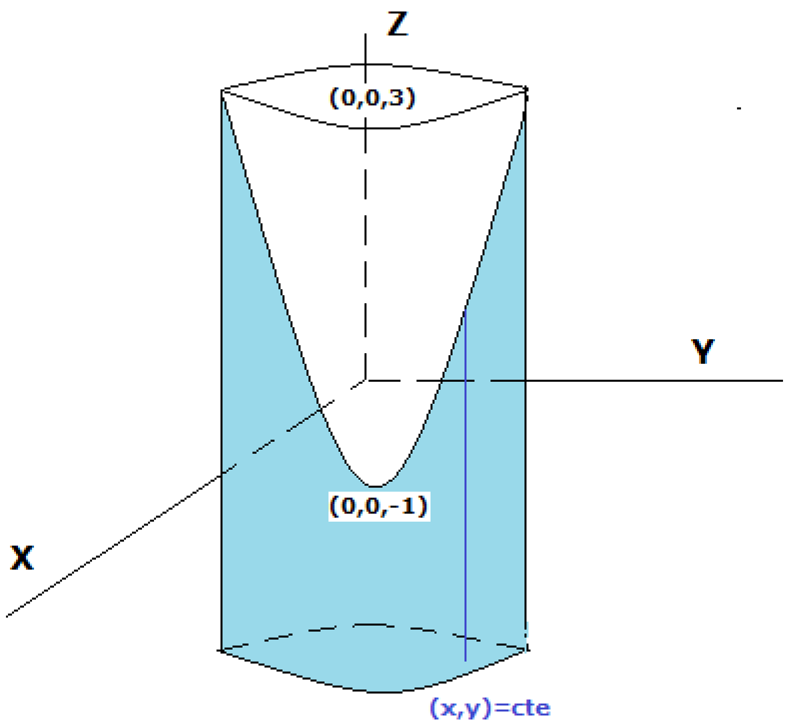
$$\int \frac{\sqrt{1+\ln x}}{x\ln x}dx; \quad \begin{cases} 1+\ln x = t^2 \\ \frac{dx}{x} = 2tdt \end{cases}$$

$$\int \frac{t.2t}{t^2-1}dt = 2\left[\int 1dt + \int \frac{1}{t^2-1}dt\right] = 2\left[t + \int \frac{1/2}{t-1}dt - \int \frac{1/2}{t+1}dt\right] = 2t + \ln\left(\frac{t-1}{t+1}\right) + cte =$$

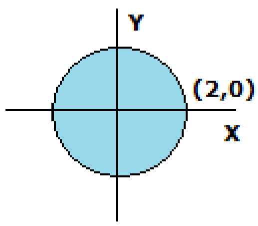
$$= 2\sqrt{1+\ln x} + \ln\left(\frac{\sqrt{1+\ln x}-1}{\sqrt{1+\ln x}+1}\right) + cte.$$

6º

$$\begin{cases} z + 1 = x^2 + y^2 \\ x^2 + y^2 = 4 \end{cases} \Rightarrow z = 3$$



$$\text{Volumen} = \iint_D dxdy \int_{-3}^{x^2+y^2-1} dz = \iint_D (x^2 + y^2 + 2)dxdy \quad \text{siendo D :}$$



Pasando a polares:

$$= \int_0^{2\pi} d\alpha \int_0^2 (r^2 + 2)rdr = 2\pi\left(\frac{r^4}{4} + r^2\right)\Bigg|_0^2 = 16\pi, \text{unidades de volumen.}$$