CÁLCULO

1º Dada la función:

$$f(x)=\begin{cases} \dfrac{x+1}{2x+1} & x\leq 0 \\ \dfrac{\operatorname{sen}x}{x} & 0<x\leq 1 \\ \ln\left(\dfrac{x-1}{x+2}\right) & x>1 \end{cases}$$

Obtener los puntos de discontinuidad y clasificarlos.

2º Calcular:

$$\int \frac{\operatorname{sen}^2 x}{1+\cos^2 x}\,dx$$

3º Invertir el orden de integración en:

$$\int_0^2 dx \int_{\frac{x^2}{2}}^{\sqrt{4x-x^2}} f(x,y)\,dy + \int_2^3 dx \int_{\sqrt{4x-x^2}}^{\frac{x^2}{2}} f(x,y)\,dy$$

4º Sea: $z=\ln(x^x+y^y)+\sqrt{\operatorname{sen}(x+y)}$.

Hallar $\dfrac{\partial z}{\partial x}+\dfrac{\partial z}{\partial y}$ en el punto $P\left(\dfrac{\pi}{4},\dfrac{\pi}{4}\right)$.

5º Hallar:

$$\iiint_V \sqrt{x^2+y^2}\,dx\,dy\,dz\,.$$

V está limitado por el paraboloide $2-z=x^2+y^2$ y el cono $z=\sqrt{x^2+y^2}$.

Solución

1º x=-1/2.

$\lim_{x\to -\frac{1}{2}^+} f(x) = \infty$, $\lim_{x\to -\frac{1}{2}^-} f(x) = -\infty \Rightarrow$ es un punto de discontinuidad

inevitable de 1ª

especie con salto infinito.

x=0.

$$\left.\begin{array}{l}\lim_{x\to 0^+}\frac{x+1}{2x+1}=1\\ \lim_{x\to 0^-}\frac{\text{sen}x}{x}=1\end{array}\right\}=1=f(0)\Rightarrow \text{la función es continua.}$$

x=1.

$$\left.\begin{array}{l}\lim_{x\to 1^-}\frac{\text{sen}x}{x}=\text{sen}1=f(1)\\ \lim_{x\to 1^+}\ln\left(\frac{x-1}{x+2}\right)=-\infty\end{array}\right\}\Rightarrow \text{ es un punto de discontinuidad}$$

inevitable de 1ª especie con salto infinito.

2º

$$\frac{\text{sen}^2x}{1+\cos^2x}=\frac{\frac{\text{sen}^2x}{\cos^2x}}{\frac{1}{\cos^2x}+1}=\frac{\text{tg}^2x}{\frac{1}{\cos^2x}+1}$$, haciendo el cambio tgx=t,

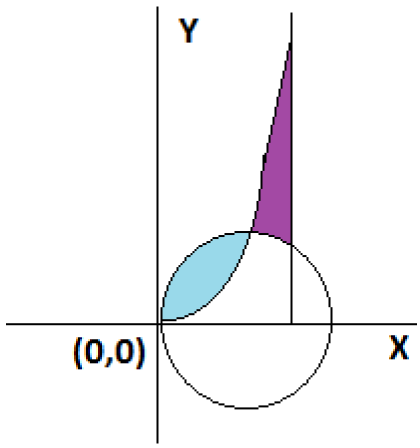
$$\int\frac{\text{sen}^2x}{1+\cos^2x}dx=\int\frac{t^2}{(t^2+2)(t^2+1)}dt;\ (*)$$

$$\frac{t^2}{(t^2+2)(t^2+1)}=\frac{A+Bt}{t^2+2}+\frac{C+Dt}{t^2+1}\Rightarrow\begin{cases}A=2\\B=0\\C=-1\\D=0\end{cases}$$

$$(*)=-\text{arctg}t+\int\frac{2}{(t^2+2)}dt=-\text{arctg}t+\int\frac{2}{2\left[\left(\frac{t}{\sqrt{2}}\right)^2+1\right]}dt=-\text{arctg}t+\sqrt{2}\,\text{arctg}\left(\frac{t}{\sqrt{2}}\right)+Cte=$$

$$=-x+\sqrt{2}\,\text{arctg}\left(\frac{\text{tg}x}{\sqrt{2}}\right)+Cte$$

3º



$$y = \sqrt{4x - x^2} \Rightarrow y^2 - 4x + x^2 = 0 \Rightarrow y^2 + (x-2)^2 = 4 .$$

$$\int_0^2 dy \int_{2-\sqrt{4-y^2}}^{\sqrt{2y}} f(x,y)dx + \int_{\sqrt{3}}^{2} dy \int_{2+\sqrt{4-y^2}}^{3} f(x,y)dx + \int_2^{\frac{9}{2}} dy \int_{\sqrt{2y}}^{3} f(x,y)dx .$$

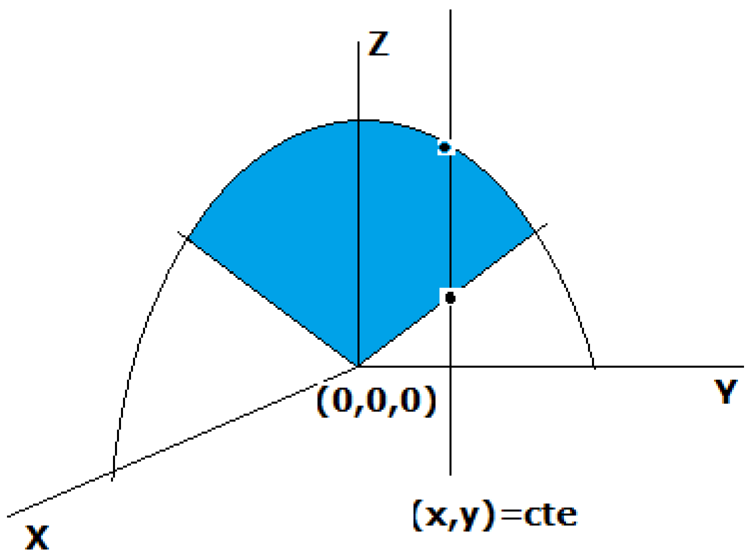
4º

$$\frac{\partial z}{\partial x} = \frac{1}{x^x + y^y}\left(x^x(\ln x + 1)\right) + \frac{1}{2\sqrt{sen(x+y)}}.\cos(x+y);$$

$$\frac{\partial z}{\partial y} = \frac{1}{x^x + y^y}\left(y^y(\ln y + 1)\right) + \frac{1}{2\sqrt{sen(x+y)}}.\cos(x+y);$$

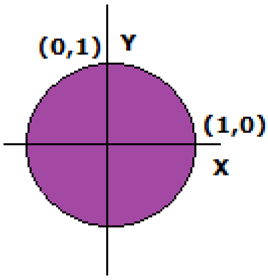
$$\frac{\partial z}{\partial x} + \frac{\partial z}{\partial y} = \ln\left(\frac{\pi}{4}\right) + 1$$

5º



$$\iint_D \sqrt{x^2+y^2}\,dx\,dy \int_{\sqrt{x^2+y^2}}^{2-x^2-y^2} dz = \iint_D \sqrt{x^2+y^2}\left(2-x^2-y^2-\sqrt{x^2+y^2}\right)dx\,dy$$

Siendo D:



Pasando a coordenadas polares:

$$\int_0^{2\pi} d\alpha \int_0^1 \left(r\,(2-r^2-r\right)r\,dr = 2\pi\left(\frac{2r^3}{3}-\frac{r^5}{5}-\frac{r^4}{4}\right)\Bigg|_0^1 = \frac{13\pi}{30}.$$